Basic measures of strain

Note: the following formulas are based on Ramsay, J.G., and Huber, M.I. (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis, Academic Press, London*, but with sign convention modified for clockwise measurement of angles and shear strains

Strain in one dimension

Extension (sometimes elongation) $e = (l-l_0)/l_0$

Stretch $S = l/l_0 = 1+e$

Quadratic elongation $\lambda = l^2/l_0^2 = (1+e)^2$

Natural strain $\varepsilon = \ln(S) = \ln(1+e) = \ln(l/l_0)$

where original length is l_0 and new length is l

Engineering shear strain $\gamma = \tan\psi$

Tensor shear strain $e_s = 0.5\tan\psi$

where angle of shear is ψ

Strain in 2 dimensions

Principal strains are designated by subscripts 1 and 3, e.g. principal elongations are $e_1 > e_3$ principal stretches are $S_1=X,\ S_3=Z$

Strain ratio $R_s = S_1/S_3$

Dilation $1+\Delta = S_1S_3$

Fundamental strain equations (Mohr circle)

For a line at an angle ϕ' from the S_1 axis,

if $\lambda' = 1/\lambda$ and $\gamma' = \gamma/\lambda$ then

$$\lambda' = \frac{\lambda_3' + \lambda_1'}{2} - \frac{\lambda_3' - \lambda_1'}{2}\cos(2\theta)$$

and

$$\gamma' = \frac{\lambda_3' - \lambda_1'}{2}\sin(2\theta)$$

If λ' is plotted against γ' these are the equations of a circle centred at $\lambda = \frac{\lambda_3' + \lambda_1'}{2}$ with radius $\frac{\lambda_3' - \lambda_1'}{2}$

Shear zones

For a simple shear zone with angle of shear ψ, shear strain γ, the extension axis S_1 is inclined to the shear zone boundary with angle θ given by:

$\gamma = \tan(\psi) = 2 / \tan(2\theta)$

Reorientation of lines from strain ellipse

For a line with initial orientation α and orientation after deformation α'

$\tan(\alpha-\theta)/\tan(\alpha'-\theta') = R_s$

where R_s is the strain ratio, θ is initial clockwise angle of S_1 from x axis; θ' is clockwise angle of S_1 from x axis after deformation.

Strain in 3 dimensions

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are $e_1 > e_2 > e_3$ Principal stretches are $S_1=X,\ S_2=Y,\ S_3=Z$

Dilation $1+\Delta = S_1S_2S_3$

Condition for plane strain: $S_2=1;\ S_1S_3=1$

Basic measures of stress

Stress Tensor

$$\mathbf{T} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

where σ_{xx} is normal stress on plane x

and σ_{xy} is shear stress on plane x parallel to axis y

Mean stress

If the principal stresses are $\sigma_1\ \sigma_2\ \sigma_3$ then:

$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

Differential stress

$\sigma_d = \sigma_1 - \sigma_3$

Deviatoric stress

$$\mathbf{T_d} = \begin{pmatrix} \sigma_{xx}-\sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}-\sigma_m & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}-\sigma_m \end{pmatrix}$$

Stress on a plane

For a plane with unit normal vector vector **x,**

$\boldsymbol{\sigma} = \mathbf{T.x}$

Resolved normal stress $\sigma_n = \boldsymbol{\sigma}.\mathbf{x}$

If σ is the magnitude of the stress on the plane, σ_n is the normal stress and σ_s is the shear stress, then:

$\sigma^2 = \sigma_n^2 + \sigma_s^2$