Vectors and Orientation data

We use an east-north-up geographic coordinate system. Axis 1 or x points east, axis 2 or y points north, and axis 3 or z points up.

Note: some textbooks use an alternative north-east-down coordinate system.

In the following, the orientation of a line is specified by plunge and trend; a plane is specified by strike and dip, using the right-hand rule.

Planes and poles

Plunge (P) and trend (T) of pole from strike (S) and dip (D) of plane

$P = 90 - D \quad T = S - 90° \text{ or } S + 270°$

Strike (S) and dip (D) of plane from plunge (P) and trend (T) of pole

D = 90-P S = T + 90° or T - 270°

Direction cosines

Vector components of a unit vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$

l,m,n are known as **direction cosines** as each is the cosine of an angle between the unit vector and a coordinate axis

Direction cosines from plunge (P) and trend (T)

$l = \sin T \cos P$

$m = \cos T \cos P$

$n = -\sin P$

Plunge (P) and trend (T) from direction cosines

$T = \tan^{-1}(l/m)$ *if m is positive or*

$T = \tan^{-1}(l/m) + 180$ *if m is negative*

$P = -\sin^{-1} n$

Plunge (P) and trend (T) from components of a general vector **a**

$T = \tan^{-1}(a_1/a_2)$ *if m is positive or*

$T = \tan^{-1}(a_1/a_2) + 180$ *if m is negative*

$P = -\sin^{-1}(z/\sqrt{(a_1^2 + a_2^2 + a_3^2)})$

Vector components of a pole to a plane from strike (S) and dip (D)

$l = -\cos S \sin D$

$m = \sin S \sin D$

$n = -\cos D$

Strike (S) and dip (D) from direction cosines of a pole

$T = \tan^{-1}(l/m) + 90$ *if m is positive or*

$T = \tan^{-1}(l/m) + 270$ *if m is negative*

$P = -\cos^{-1} n$

Note: in Excel, the *tan*$^{-1}$ formulas can conveniently be expressed =DEGREES(ATAN2(M,L))

Statistics of orientation data

Statistics for a set of unit vectors: $\mathbf{\hat{a}_1}, \mathbf{\hat{a}_2}, \mathbf{\hat{a}_3} \ldots \mathbf{\hat{a}_n}$

Vector sum $\mathbf{R} = (\mathbf{\hat{a}}_1 + \mathbf{\hat{a}}_2 + \mathbf{\hat{a}}_3 + \ldots\ldots \mathbf{\hat{a}}_n)$

Resultant $R = |\mathbf{R}|$

Vector mean $\mathbf{r} = \mathbf{\bar{R}} = \mathbf{R} / n$

Mean resultant $r = |\mathbf{r}| = R/n$

Direction cosine matrix of a set of unit vectors

The matrix $\begin{pmatrix} \sum l^2 & \sum lm & \sum ln \\ \sum lm & \sum m^2 & \sum mn \\ \sum ln & \sum mn & \sum n^2 \end{pmatrix}$

has eigenvectors $\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e_3}$, and eigenvalues $e_1 \le e_2 \le e_3$ Vectors and plate motion