Vector operations

Representation of vectors

In the following, vector **a** is represented $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

with components in directions east, north and up.

In printed text, vectors are bold. In handwritten text, vectors are underlined or overscored with an arrow: $\underline{a}$ $\vec{a}$

Magnitude of vector **a** is

$$a \text{ or } |\mathbf{a}| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$$

Unit vector in same direction as **a**

$$\hat{\mathbf{a}} = \begin{pmatrix} a_1 / |\mathbf{a}| \\ a_2 / |\mathbf{a}| \\ a_3 / |\mathbf{a}| \end{pmatrix}$$

Basic vector operations

Vector addition

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$

Vector dot product

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Vector dot product is a scalar **a.b** = $ab\cos\theta$ where θ is the angle between **a** and **b**

Vector cross product

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - b_2 a_3 \\ a_3 b_1 - b_3 a_1 \\ a_1 b_2 - b_1 a_2 \end{pmatrix}$$

Vector cross product is a vector with magnitude $ab\sin\theta$ where θ is the angle between **a** and **b.** $\mathbf{a} \times \mathbf{b}$ is perpendicular to **a** and **b**. Vectors **a**, **b** and $\mathbf{a} \times \mathbf{b}$ form a right-handed set.

Basic matrix algebra

Symmetric matrix

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \text{ or } a_{ij} = a_{ji}$$

Skew symmetric matrix

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ -b & d & e \\ -c & -e & f \end{pmatrix} \text{ or } a_{ij} = -a_{ji}$$

Transpose of a matrix

$$\mathbf{A}^{\mathrm{T}} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

or $a^T_{ij} = a_{ji}$

Matrix addition (shown for 2 x 2 matrix)

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$$

or in general $a_{ij} + b_{ij} = c_{ij}$

Matrix multiplication (shown for 2 x 2 matrix)

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

or, in general $a_{ip}b_{pj} = c_{ij}$

The unit matrix or Kronecker's $\boldsymbol{\delta}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ or } \delta_{ij} \text{ or } \boldsymbol{\delta}$$

Determinant of a 2 x 2 matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of a 3 x 3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Eigenvectors of a matrix **A** are solutions x to:

$$\mathbf{A}.\mathbf{x} = k\mathbf{x}$$

where k is an eigenvalue associated with eigenvector **x**. For a symmetric matrix **A** the eigenvectors are mutually perpendicular.