Basic measures of strain

Strain in one dimension

Extension (sometimes elongation) $e = (l - l_0)/l_0$

Stretch $s = l/l_0 = 1+e$

Quadratic elongation $\lambda = l^2/l_0^2 = (1+e)^2$

Natural strain $\varepsilon = ln(s) = ln(1+e) = ln(l/l_0)$

where original length is l_0 and new length is l

Engineering shear strain $\gamma = \tan \psi$

Tensor shear strain $e_s = 0.5 \tan \psi$

where angle of shear is ψ

*Strain in 2 dimensions**

Principal strains are designated by subscripts 1 and 3, e.g. principal elongations are $e_1 > e_3$

principal stretches are $s_1 = X$, $s_3 = Z$

Strain ratio $R_s = s_1/s_3$

Dilation $1+\Delta = s_1 s_3$

Fundamental strain equations (Mohr circle)

For a line at an angle θ from the S_1 axis,

if $\lambda' = 1/\lambda$ and $\gamma' = \gamma/\lambda$ *then*

$$\lambda' = \frac{\lambda'_3 + \lambda'_1}{2} - \frac{\lambda'_3 - \lambda'_1}{2}\cos(2\theta)$$

and

$$\gamma' = \frac{\lambda'_3 - \lambda'_1}{2}\sin(2\theta)$$

If λ' is plotted against γ' these are the equations of a circle centred at $\lambda = \frac{\lambda'_3 + \lambda'_1}{2}, \gamma = 0$

with radius $\frac{\lambda'_3 - \lambda'_1}{2}$

Matrix F (deformation gradient tensor) describes relation between points in undeformed and deformed state

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

where (x_0, y_0) is original location and (x,y) is final location.

Alternatively $\mathbf{Fx_0} = \mathbf{x}$

Combining deformations: Matrix C for two deformations B followed by A

C=**AB**

Dilation

$1+\Delta = ad-bc$

Rotation of strain axes

Angle ω between deformed and undeformed states

$\tan\omega = (b-c)/(a+d)$

Shear zones

For a simple shear zone with angle of shear ψ, shear strain γ, the extension axis S_1 is inclined to the shear zone boundary with angle θ given by:

$\gamma = \tan(\psi) = 2 / \tan(2\theta)$

Reorientation of lines from strain ellipse

For a line with initial orientation α and orientation after deformation α'

$\tan(\alpha - \theta)/\tan(\alpha' - \theta') = R_s$

where R_s is the strain ratio, θ is initial clockwise angle of s_1 from x axis; θ' is final clockwise angle of s_1 from x axis.

*Strain in 3 dimensions**

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are $e_1 > e_2 > e_3$

Principal stretches are $s_1 = X$, $s_2 = Y$, $s_3 = Z$

Dilation $1+\Delta = s_1 s_2 s_3$

Condition for plane strain: $s_2 = 1$; $s_1 s_3 = 1$

Strain ratios for Flinn plot

$a = s_1/s_2$

$b = s_2/s_3$

Shape parameter $k = a/b$

*Note: the formulas are based on Ramsay, J.G., and Huber, M.I. (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis, Academic Press, London*, but with sign convention modified for clockwise measurement of angles and shear strains