Basic measures of stress

Stress Tensor

$$\mathbf{T} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

where σ_{12} is shear stress acting parallel to axis 2 on the plane perpendicular to axis 1

T is symmetric (i.e. $\sigma_{xy} = \sigma_{yx}$)

Principal stresses

$\sigma_1 > \sigma_2 > \sigma_3$ are the normal stresses that operate parallel to the eigenvectors of **T** (the stress axes). If the stress axes coincide with the coordinate axes then

$$\mathbf{T} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

Mean stress

If the magnitudes of the principal stresses are σ_1 σ_2 σ_3 then:

mean stress $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

Differential stress

Differential stress $\sigma_d = \sigma_1 - \sigma_3$

Maximum shear stress = $\sigma_d/2$

Deviatoric stress

$$\mathbf{T_d} = \begin{pmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{pmatrix}$$

Stress on a plane

For a plane with unit normal vector vector **x**,

$$\boldsymbol{\sigma} = \mathbf{T.x}$$

Resolved normal stress $\sigma_n = \boldsymbol{\sigma}.\mathbf{x}$

Resolved shear stress $\sigma_s^2 = \sqrt{(\sigma^2 - \sigma_n^2)}$

Where σ, σ_n, σ_s are the magnitudes of the resolved shear stress, and its normal and shear components.

Effective stress

$$\text{Effective stress} = \begin{pmatrix} \sigma_{11}-P_f & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-P_f & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-P_f \end{pmatrix}$$

where P_f = pore fluid pressure

Stress states

If ρ is the average density of overlying material, g is the acceleration due to gravity, and z is depth, and ν is Poisson's ratio

Lithostatic/hydrostatic stress

$$\sigma_1 = \sigma_2 = \sigma_3 = \rho g z$$

Uniaxial-strain reference state

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_z = \frac{\nu}{1-\nu}\rho g z$$

Stress-strain relationships

(Elastic/brittle behaviour)

Elastic strain

Linear stress $\sigma_n = E.e$

Poisson's ratio $\nu = -e_1/e_3$

shear stress $\sigma_s = G.\gamma$

mean stress $\sigma_m = -K\Delta$

where E = Young's modulus of elasticity;

G = shear modulus of elasticity;

K = bulk modulus of elasticity;

e_1, e_3 are principal extensions of sample under uniaxial compression.

Relationships between the elastic moduli

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Brittle Failure

Coulomb fracture criterion

$\sigma_s = C + \sigma_n \tan\phi$

where C is a constant and ϕ is the angle of internal friction.

Angle between pole to failure plane and σ_1

$\theta = 45° + \phi/2$

"Byerlee's law" for movement on existing fractures

$\sigma_s = 0.85\sigma_n$ ($\sigma_n < 200$ MPa)

$\sigma_s = 50$ MPa $+ 0.60\sigma_n$ ($\sigma_n > 200$ MPa)