

Basic measures of stress

Stress Tensor

$$\mathbf{T} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

where σ_{12} is shear stress acting parallel to axis 2 on the plane perpendicular to axis 1

\mathbf{T} is symmetric (i.e. $\sigma_{xy} = \sigma_{yx}$)

Principal stresses

$\sigma_1 > \sigma_2 > \sigma_3$ are the normal stresses that operate parallel to the eigenvectors of \mathbf{T} (the stress axes).
If the stress axes coincide with the coordinate axes then

$$\mathbf{T} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

Mean stress

If the magnitudes of the principal stresses are σ_1 σ_2 σ_3 then:

mean stress $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

Differential stress

Differential stress $\sigma_d = \sigma_1 - \sigma_3$

Maximum shear stress = $\sigma_d/2$

Deviatoric stress

$$\mathbf{T}_d = \begin{pmatrix} \sigma_{11} - \sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \sigma_m \end{pmatrix}$$

Stress on a plane

For a plane with unit normal vector $\hat{\mathbf{a}}$,

$$\boldsymbol{\sigma} = \mathbf{T} \cdot \hat{\mathbf{a}}$$

Resolved normal stress $\sigma_n = \boldsymbol{\sigma} \cdot \hat{\mathbf{a}}$

Resolved shear stress $\sigma_s^2 = \sqrt{(\boldsymbol{\sigma}^2 - \sigma_n^2)}$

Where σ , σ_n , σ_s are the magnitudes of the resolved shear stress, and its normal and shear components.

Effective stress

$$\text{Effective stress} = \begin{pmatrix} \sigma_{11} - P_f & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - P_f & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - P_f \end{pmatrix}$$

where P_f = pore fluid pressure

Stress states

If ρ is the average density of overlying material, g is the acceleration due to gravity, and z is depth, and ν is Poisson's ratio

Lithostatic/hydrostatic stress

$$\sigma_1 = \sigma_2 = \sigma_3 = \rho g z$$

Uniaxial-strain reference state

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu} \sigma_z = \frac{\nu}{1-\nu} \rho g z$$

Stress-strain relationships

(Elastic/brittle behaviour)

Elastic strain

Linear stress $\sigma_n = E \cdot e$

Poisson's ratio $\nu = -e_1/e_3$

shear stress $\sigma_s = G \cdot \gamma$

mean stress $\sigma_m = -K\Delta$

where E = Young's modulus of elasticity;

e = extension

e_1 , e_3 are principal extensions of sample under uniaxial compression.

γ = shear strain

G = shear modulus of elasticity

$$G = \frac{E}{2(1+\nu)}$$

K = bulk modulus of elasticity;

$$K = \frac{E}{3(1-2\nu)}$$

Brittle Failure

Coulomb fracture criterion

$$\sigma_s = C + \sigma_n \tan \phi$$

where C is a constant and ϕ is the angle of internal friction.

Angle between pole to failure plane and σ_1

$$\theta = 45^\circ + \phi/2$$

"Byerlee's law" for movement on existing fractures

$$\sigma_s = 0.85 \sigma_n \quad (\sigma_n < 200 \text{ MPa})$$

$$\sigma_s = 50 \text{ MPa} + 0.60 \sigma_n \quad (\sigma_n > 200 \text{ MPa})$$