# Vectors and Orientation data

We use an east-north-up geographic coordinate system. Axis 1 or x points east, axis 2 or y points north, and axis 3 or z points up.

Note: some textbooks use an alternative north-east-down coordinate system.

In the following, the orientation of a line is specified by plunge and trend; a plane is specified by strike and dip, using the right-hand rule.

## Planes and poles

Plunge (P) and trend (T) of pole from strike (S) and dip (D) of plane

$P = 90\text{-}D \quad T = S - 90° \text{ or } S + 270°$

Strike (S) and dip (D) of plane from plunge (P) and trend (T) of pole

$D = 90\text{-}P \quad S = T + 90° \text{ or } T - 270°$

## Direction cosines from field orientations

Vector components of a unit vector $\hat{\mathbf{a}} = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$

*l,m,n* are known as **direction cosines** as each is the cosine of an angle between the unit vector and a coordinate axis

The following formulas yield downward directed vectors when dip is positive from 0-90° (the normal geological convention)

Direction cosines from plunge (P) and trend (T)

$l = \sin T \cos P$

$m = \cos T \cos P$

$n = -\sin P$

Vector components of a pole to a plane from strike (S) and dip (D)

$l = -\cos S \sin D$

$m = \sin S \sin D$

$n = -\cos D$

## Field orientations from direction cosines

These calculations are a little more complex, as the starting vector may be directed either upwards or downwards, whereas geologists typically measure plunge and dip in the downward direction. The simplest way to deal with this is to **reverse any vector that has positive** *n* **(**or $a_3$**)** by first changing the sign of all three components: *l, m, n, (*or $a_1$, $a_2$, $a_3$*)*.

Plunge (P) and trend (T) from direction cosines of a **down-directed unit vector**.

$T = \tan^{-1}(l/m)$ *if m is positive or*

$T = \tan^{-1}(l/m) + 180$ *if m is negative*

$P = -\sin^{-1} n$

Plunge (P) and trend (T) from components of a general **down-directed** vector **a**

$T = \tan^{-1}(a_1/a_2)$ *if m is positive or*

$T = \tan^{-1}(a_1/a_2) + 180$ *if m is negative*

$P = -\sin^{-1}(a_3 / \sqrt{(a_1^2 + a_2^2 + a_3^2)})$

Strike (S) and dip (D) from direction cosines of a **down-directed unit pole vector**.

$S = \tan^{-1}(l/m) + 90$ *if m is positive or*

$S = \tan^{-1}(l/m) + 270$ *if m is negative*

$D = -\cos^{-1}(n)$

Note: in Excel, the $\tan^{-1}$ formulas can conveniently be expressed =DEGREES(ATAN2(M,L))

## Statistics of orientation data

Statistics for a set of unit vectors: $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \hat{\mathbf{a}}_3 \ldots \hat{\mathbf{a}}_n$

Vector sum $\mathbf{R} = (\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2 + \hat{\mathbf{a}}_3 + \ldots\ldots \hat{\mathbf{a}}_n)$ or

$$\mathbf{R} = \begin{pmatrix} \sum l \\ \sum m \\ \sum n \end{pmatrix}$$

Resultant $R = |\mathbf{R}|$

Vector mean $\mathbf{r} = \bar{\mathbf{R}} = \mathbf{R} / n$

Mean resultant $r = |\mathbf{r}| = R/n$

Direction cosine matrix of a set of unit vectors

The matrix $\begin{pmatrix} \sum l^2 & \sum lm & \sum ln \\ \sum lm & \sum m^2 & \sum mn \\ \sum ln & \sum mn & \sum n^2 \end{pmatrix}$

has eigenvectors $\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{e_3}$, and eigenvalues $e_1 \geq e_2 \geq e_3$

 