Finite strain

Strain in one dimension

For original length l_0, new length l angle of shear ψ

Extension (sometimes elongation) $e = (l-l_0)/l_0$

Stretch $s = l/l_0 = 1+e$

Quadratic elongation $\lambda = l^2/l_0^2 = (1+e)^2$

Natural strain $\varepsilon = ln(s) = ln(1+e) = ln(l/l_0)$

Engineering shear strain $\gamma = tan\ \psi$

Tensor shear strain $e_s = 0.5\ tan\ \psi$

*Strain in 2 dimensions**

Principal strains are designated by subscripts 1 and 3, e.g. principal elongations are $e_1 > e_3$ principal stretches are $s_1=X,\ s_3=Z$

Strain ratio $R_s = s_1/s_3$

Dilation $1+\Delta = s_1 s_3$

Fundamental strain equations (Mohr circle for strain)

For a line at an angle θ from the s_1 axis,

if $\lambda' = 1/\lambda$ and $\gamma' = \gamma/\lambda$ *then*

$$\lambda' = \frac{\lambda_3' + \lambda_1'}{2} - \frac{\lambda_3' - \lambda_1'}{2}\cos(2\theta)$$

$$\gamma' = \frac{\lambda_3' - \lambda_1'}{2}\sin(2\theta)$$

If λ' is plotted against γ' these are the equations of a circle centred at

$$\lambda = \frac{\lambda_3' + \lambda_1'}{2}, \gamma = 0 \text{ with radius } \frac{\lambda_3' - \lambda_1'}{2}$$

Deformation gradient tensor F

Describes relation between points in undeformed and deformed state $\mathbf{Fx_0} = \mathbf{x}$ or

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

where $\mathbf{x_0}=(x_0,y_0)$ is original location of a point and $\mathbf{x}=(x,y)$ is final location and $a\ b\ c\ d$ are components of **F**.

Reciprocal deformation gradient tensor $\mathbf{F}^{-1}$

$$\mathbf{F}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} d/_{ad-bc} & -b/_{ad-bc} \\ -c/_{ad-bc} & a/_{ad-bc} \end{pmatrix}$$

This 'undoes' the deformation: $\mathbf{F^{-1}x = x_0}$

Combining deformations: Matrix $\mathbf{F_c}$ for two deformations $\mathbf{F_a}$ followed by $\mathbf{F_b}$ is obtained by matrix multiplication:

$\mathbf{F_c = F_b F_a}$

Displacement gradient tensor

$$\mathbf{J} = (\mathbf{F} - \delta) = \begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix}$$

describes relation of displacement to position of points in undeformed state: $\mathbf{Jx_0 = x - x_0}$

Strain tensor

$$\mathbf{E} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} a-1 & 0.5(b+c) \\ 0.5(b+c) & d-1 \end{pmatrix}$$

describes the non-rotational part of **J**

Dilation

$1+\Delta = ad-bc = |\mathbf{F}|$

Rotation of strain axes

Angle ω between deformed and undeformed states

$tan\omega = (b-c)/(a+d)$

Reorientation of lines from strain ellipse

If θ = initial orientation of a line relative to s_1 and θ' is orientation after deformation, then

$tan(\theta)/tan(\theta') = R_s$ where R_s is the strain ratio

Simple shear zones

If γ=shear strain, orientation θ of extension axis s_1 to shear-zone boundary is given by: $\tan(2\theta) = 2/\gamma$

*Strain in 3 dimensions**

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are $e_1 > e_2 > e_3$ Principal stretches are $s_1=X,\ s_2=Y,\ s_3=Z$

Dilation $1+\Delta = s_1 s_2 s_3$

Condition for plane strain: $s_2=1;\ s_1 s_3=1$

Strain ratios for Flinn plot

$a = s_1/s_2$

$b = s_2/s_3$

Shape parameter $k = a/b$

*Note: the formulas are based on Ramsay, J.G., and Huber, M.I. (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis, Academic Press, London*, but with sign convention modified for clockwise measurement of angles and shear strains