

Ductile deformation flow laws

Plastic (Von Mises) Creep

$$\sigma_d = C \text{ (constant)}$$

where σ_d is differential stress.

Viscous (Newtonian) flow

$$\text{Strain rate } \dot{\epsilon} \propto \sigma_d$$

where σ_d is differential stress. Alternatively:

$$\sigma_d = \eta \dot{\epsilon}$$

where η = viscosity.

Specific equation for diffusional creep

$$\dot{\epsilon} = A \sigma_d \exp\left(-E^*/RT\right) d^{-r}$$

where d is the grain size and r is 2 for grain-boundary diffusion and 3 for volume diffusion

Power Law Creep

$$\dot{\epsilon} \propto \sigma_d^n$$

where k and n are constants

or more specifically

$$\dot{\epsilon} = A \sigma_d^n \exp\left(-E^*/RT\right)$$

where A and n are constants for the material, E^* is the activation energy, R is the gas constant and T is the absolute temperature.

Exponential Creep

$$\dot{\epsilon} \propto \exp(\sigma_d)$$

or more specifically:

$$\dot{\epsilon} = A \exp(\sigma_d) \exp\left(-E^*/RT\right)$$

where A is a constant for the material, E^* is the activation energy, R is the gas constant and T is the absolute temperature

Progressive strain and flow

$$\text{Longitudinal strain rate } \dot{\epsilon} = \frac{de}{dt}$$

$$\text{Shear strain rate } \dot{\gamma} = \frac{d\gamma}{dt}$$

$$\text{Velocity gradient matrix } \mathbf{L} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$$

$$\text{or (in 2 dimensions) } \mathbf{L} = \begin{pmatrix} L_{11} & L_{13} \\ L_{31} & L_{33} \end{pmatrix}$$

Velocity at any point x is given by $\mathbf{v} = \mathbf{Lx}$

$$\text{or } \begin{pmatrix} v_1 \\ v_3 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{13} \\ L_{31} & L_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$$

Stretching and rotation components of L

$$\mathbf{L} = \mathbf{S} + \mathbf{W} = \begin{pmatrix} \dot{\epsilon}_{11} & \frac{1}{2}\dot{\gamma} \\ \frac{1}{2}\dot{\gamma} & \dot{\epsilon}_{33} \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}\dot{\gamma} \\ -\frac{1}{2}\dot{\gamma} & 0 \end{pmatrix}$$

Kinematic vorticity number:

$$W_k = \text{rotation rate/distortion rate} = \cos(\alpha)$$

where α is the angle between the eigenvectors of \mathbf{L}

Simple shear zones

If γ = shear strain, orientation θ of finite extension axis s_1 to shear-zone boundary is given by: $\tan(2\theta) = 2/\gamma$