Vector operations

Coordinate systems

For local structural observations, we use a geographic coordinate system. Axis x points east, axis y points north, and axis z points up. Note that other coordinate systems are possible.

For calculations dealing with the whole Earth we use a coordinate system in which the origin is at the centre of the Earth. Axis x = points towards the intersection of the equator and the Greenwich meridian, y is towards the intersection of the equator and 90°E, and z = is towards the north pole.

Representation of vectors

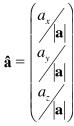
In the following, vector **a** is represented $\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$

with components in directions east, north and up.

In printed text, vectors are bold. In handwritten text, vectors are underlined thus **a**

Magnitude of vector **a** is represented by *a* or $|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Unit vector in same direction as a



Basic vector operations

Vector addition

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} + \begin{pmatrix} b_x \\ b_y \\ b_x \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{pmatrix}$$

Vector dot product $\mathbf{a}.\mathbf{b} = ab\cos\theta$ where θ is the angle between **a** and **b**

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \bullet \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$$

Vector cross product

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - b_y a_z \\ a_z b_x - b_z a_x \\ a_x b_y - b_x a_y \end{pmatrix}$$

Vectors and Orientation data

Planes and poles

Plunge (P) and trend (T) of pole from strike (S) and dip (D) of plane P = 90-D $T = S - 90^{\circ} \text{ or } S + 270^{\circ}$ Strike (S) and dip (D) of plane from plunge (P) and trend (T) of pole D = 90-P $S = T + 90^{\circ}$ or $T - 270^{\circ}$ Direction cosines Vector components of a unit vector (direction cosines) (l,m,n) from plunge (P) and trend (T) l = sin T cos P $m = \cos T \cos P$ n = -sin PPlunge (P) and trend (T) from direction cosines (l,m,n) (components of unit vector) $T = tan^{-1}(l/m)$ if m is positive or $T = tan^{-1}(l/m) + 180$ if m is negative $P = -sin^{-1}n$ Plunge (P) and trend (T) from components of a general vector (x,y,z) $T = tan^{-1}(x/y)$ if m is positive or $T = tan^{-1}(x/y) + 180 \text{ if } m \text{ is negative}$ $P = -sin^{-1}(z/\sqrt{(x^2+y^2+z^2)})$ Vector components of a pole to a plane (direction cosines) (1,m,n) from strike (S) and dip (D) $l = -\cos S \sin D$ m = sin S sin D $n = -\cos D$ Strike (S) and dip (D) from direction cosines of a pole (l,m,n) $T = tan^{-1}(l/m) + 90$ if m is positive or $T = tan^{-1}(l/m) + 270$ if m is negative $P = -cos^{-1} n$ Statistics of orientation data Statistics for a set of unit vectors: $\hat{a}_1, \hat{a}_2, \hat{a}_3 \dots \hat{a}_n$ Vector sum $\mathbf{R} = (\mathbf{\hat{a}}_1 + \mathbf{\hat{a}}_2 + \mathbf{\hat{a}}_3 + \dots \mathbf{\hat{a}}_n)$ Resultant $R = |\mathbf{R}|$ Vector mean $\mathbf{r} = \overline{\mathbf{R}} = \mathbf{R}/n$ Mean resultant $r = |\mathbf{r}| = R/n$ Direction cosine matrix of a set of unit vectors (∇)

The matrix
$$\begin{bmatrix} \sum l^2 & \sum lm & \sum ln \\ \sum lm & \sum m^2 & \sum mn \\ \sum ln & \sum mn & \sum n^2 \end{bmatrix}$$

has eigenvectors E_1 , E_2 , E_3 , and eigenvalues $E_1 \leq E_2 \leq E_3$