Basic matrix algebra

Symmetric matrix

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \text{ or } a_{ij} = a_{ji}$$

Skew symmetric matrix

$$\begin{pmatrix} a & b & c \\ -b & d & e \\ -c & -e & f \end{pmatrix} \text{ or } a_{ij} = -a_{ji}$$

Transpose of a matrix

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \mathbf{a}^T$$

or $a^T_{ij} = a_{ji}$

Matrix addition (shown for 2 x 2 matrix)

$$\begin{pmatrix} a_{xx} & a_{xz} \\ a_{zx} & a_{zz} \end{pmatrix} + \begin{pmatrix} b_{xx} & b_{xz} \\ b_{zx} & b_{zz} \end{pmatrix} = \begin{pmatrix} a_{xx}+b_{xx} & a_{xz}+b_{xz} \\ a_{zx}+b_{zx} & a_{zz}+b_{zz} \end{pmatrix}$$

or in general $a_{ij} + b_{ij} = c_{ij}$

Matrix multiplication (shown for 2 x 2 matrix)

$$\begin{pmatrix} a_{xx} & a_{xz} \\ a_{zx} & a_{zz} \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xz} \\ b_{zx} & b_{zz} \end{pmatrix} = \begin{pmatrix} a_{xx}b_{xx}+a_{xz}b_{zx} & a_{xx}b_{xz}+a_{xz}b_{zz} \\ a_{zx}b_{xx}+a_{zz}b_{zx} & a_{zx}b_{xz}+a_{zz}b_{zz} \end{pmatrix}$$

or, in general $a_{ip}b_{pj} = c_{ij}$

The unit matrix or Kronecker's δ

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ or } \delta_{ij} \text{ or } \boldsymbol{\delta}$$

Determinant of a 2 x 2 matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of a 3 x 3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Eigenvectors of a matrix **A** are solutions x to:

$$\mathbf{A}.\mathbf{x} = k\mathbf{x}$$

where k is an eigenvalue associated with eigenvector **x**. For a symmetric matrix **A** the eigenvectors are mutually perpendicular.

Basic measures of stress

Stress Tensor

$$\mathbf{T} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

where σ_{xx} is normal stress on plane x
and σ_{xy} is shear stress on plane x parallel to axis y
T is symmetric (i.e. $\sigma_{xy} = \sigma_{yx}$)

Principal stresses

$\sigma_1 > \sigma_2 > \sigma_3$ are the normal stresses that operate parallel to the eigenvectors of **T** (the stress axes).

Mean stress

If the magnitudes of the principal stresses are σ_1 σ_2 σ_3 then:

$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

Differential stress

$\sigma_d = \sigma_1 - \sigma_3$

Maximum shear stress = $\sigma_d/2$

Deviatoric stress

$$\mathbf{T_d} = \begin{pmatrix} \sigma_{xx}-\sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}-\sigma_m & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}-\sigma_m \end{pmatrix}$$

Stress on a plane

For a plane with unit normal vector vector **x,**

$\boldsymbol{\sigma} = \mathbf{T}.\mathbf{x}$

Resolved normal stress $\sigma_n = \boldsymbol{\sigma}.\mathbf{x}$

If σ is the magnitude of the stress on the plane, σ_n is the magnitude of the normal stress and σ_s is the magnitude of the shear stress, then:

$\sigma^2 = \sigma_n^2 + \sigma_s^2$