# Basic measures of strain

Note: the following formulas are based on Ramsay, J.G., and Huber, M.I. (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis, Academic Press, London*, but with sign convention modified for clockwise measurement of angles and shear strains

### Strain in one dimension

**Extension** (sometimes elongation) $e = (l-l_0)/l_0$

**Stretch** $S = l/l_0 = 1+e$

**Quadratic elongation** $\lambda = l^2/l_0^2 = (1+e)^2$

**Natural strain** $\varepsilon = ln(S) = ln(1+e) = ln(l/l_0)$

where original length is $l_0$ and new length is $l$

**Engineering shear strain** $\gamma = tan\ \psi$

**Tensor shear strain** $e_s = 0.5\ tan\ \psi$

where angle of shear is $\psi$

### Strain in 2 dimensions

Principal strains are designated by subscripts 1 and 3, e.g. principal elongations are $e_1 > e_3$ principal stretches are $S_1=X,\ S_3=Z$

**Strain ratio** $R_s = S_1/S_3$

**Dilation** $1+\Delta = S_1S_3$

**Fundamental strain equations (Mohr circle)**

For a line at an angle $\theta$ from the $S_1$ axis,

if $\lambda' = 1/\lambda$ and $\gamma' = \gamma/\lambda$ *then*

$$\lambda' = \frac{\lambda_3' + \lambda_1'}{2} - \frac{\lambda_3' - \lambda_1'}{2}\cos(2\theta)$$

and

$$\gamma' = \frac{\lambda_3' - \lambda_1'}{2}\sin(2\theta)$$

If $\lambda'$ is plotted against $\gamma'$ these are the equations of a circle centred at $\lambda = \frac{\lambda_3' + \lambda_1'}{2}$ with radius $\frac{\lambda_3' - \lambda_1'}{2}$

**Shear zones**

For a simple shear zone with angle of shear $\psi$, shear strain $\gamma$, the extension axis $S_1$ is inclined to the shear zone boundary with angle $\theta$ given by:

$\gamma = \tan(\psi) = 2 / \tan(2\theta)$

**Reorientation of lines from strain ellipse**

For a line with initial orientation $\alpha$ and orientation after deformation $\alpha'$

$tan(\alpha-\theta)/tan(\alpha'-\theta') = R_s$

where $R_s$ is the strain ratio, $\theta$ is initial clockwise angle of $S_1$ from x axis; $\theta'$ is clockwise angle of $S_1$ from $x$ axis after deformation.

### Deformation matrix (deformation gradient tensor)

**Matrix F** describes relation between points in undeformed and deformed state

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

where $(x_0,y_0)$ is original location and $(x,y)$ is final location.

Alternatively $\mathbf{Fx_0} = \mathbf{x}$

**Reciprocal deformation matrix $\mathbf{F^{-1}}$**

$$\mathbf{F}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} d/(ad-bc) & -b/(ad-bc) \\ -c/(ad-bc) & a/(ad-bc) \end{pmatrix}$$

This has the effect of 'undoing' the deformation:

$\mathbf{F^{-1}x = x_0}$

**Combining deformations**: Matrix C for two deformations B followed by A

**C=AB**

### Some deformation matrices

Matrix for **pure dilation** $\Delta$ $\begin{pmatrix} 1+\Delta & 0 \\ 0 & 1+\Delta \end{pmatrix}$

Matrix for clockwise **rotation** $\omega$ $\begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}$

Matrix for **pure strain** parallel to x and y $\begin{pmatrix} S_x & 0 \\ 0 & S_z \end{pmatrix}$

Matrix for general pure strain $\begin{pmatrix} a & b \\ b & d \end{pmatrix}$

Matrix for **pure shear** parallel to x and y $\begin{pmatrix} S & 0 \\ 0 & 1/S \end{pmatrix}$

Matrix for **simple shear** parallel to x-axis $\begin{pmatrix} 1 & \gamma \\ 0 & 1 \end{pmatrix}$

### Displacement gradient tensor

$$\mathbf{J} = (\mathbf{F} - \boldsymbol{\delta}) = \begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix}$$

describes relation between points in undeformed state and displacement:

$\mathbf{Jx_0 = x - x_0}$

### Strain tensor

$$\mathbf{E} = \begin{pmatrix} e_{xx} & e_{xz} \\ e_{zx} & e_{zz} \end{pmatrix} = \begin{pmatrix} a-1 & 0.5(b+c) \\ 0.5(b+c) & d-1 \end{pmatrix}$$

describes the non-rotational part of displacement gradient.