

Stress-strain relationships

Elastic strain

Linear stress $\sigma_n = E \cdot e$

Poisson's ratio $\nu = -e_1/e_3$

shear stress $\sigma_s = G \cdot \gamma$

mean stress $\sigma_m = -K\Delta$

where E = Young's modulus of elasticity;

G = shear modulus of elasticity;

K = bulk modulus of elasticity;

e_1, e_3 are principal extensions of sample under uniaxial compression.

Relationships between the elastic moduli

$$G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)}$$

Brittle Failure

Coulomb fracture criterion

$$\sigma_s = C + \sigma_n \tan\phi$$

where C is a constant and ϕ is the angle of internal friction.

"Byerlee's law" for movement on existing fractures

$$\sigma_s = 0.85 \sigma_n \quad (\sigma_n < 200 \text{ MPa})$$

$$\sigma_s = 50 + 0.60 \sigma_n \quad (\sigma_n > 200 \text{ MPa})$$

Strain in 3 dimensions

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are $e_1 > e_2 > e_3$

Principal stretches are $S_1=X, S_2=Y, S_3=Z$

$$\text{Dilation } 1+\Delta = S_1 S_2 S_3$$

$$\text{Condition for plane strain: } S_2=1; S_1 S_3=1$$

Strain ratios for Flinn plot

$$a = S_1/S_2$$

$$b = S_2/S_3$$

Shape parameter $k = a/b$

Viscous (Newtonian) flow

Strain rate $\dot{\epsilon} \propto \sigma_d$

where σ_d is differential stress. Alternatively:

$$\sigma_d = \eta \dot{\epsilon}$$

where η = viscosity.

Specific equation for diffusional creep

$$\dot{\epsilon} = A \sigma_d \exp\left(-E^*/RT\right) d^{-r}$$

where d is the grain size and r is 2 for grain-boundary diffusion and 3 for volume diffusion

Plastic (Von Mises) Creep

$$\sigma_d = C \quad (\text{constant})$$

Power Law Creep

$\dot{\epsilon} \propto \sigma_d^n$ where k and n are constants

or more specifically

$$\dot{\epsilon} = A \sigma_d^n \exp\left(-E^*/RT\right)$$

where A and n are constants for the material, E* is the activation energy, R is the gas constant and T is the absolute temperature.

Exponential Creep

$\dot{\epsilon} \propto \exp(\sigma_d)$ where k and n are constants

or more specifically:

$$\dot{\epsilon} = A \exp(\sigma_d) \exp\left(-E^*/RT\right)$$

where A is a constant for the material, E* is the activation energy, R is the gas constant and T is the absolute temperature