Vector operations

Coordinate systems

For local structural observations, we use a geographic coordinate system. Axis x points east, axis y points north, and axis z points up. Note that other coordinate systems are possible.

For calculations dealing with the whole Earth we use a coordinate system in which the origin is at the centre of the Earth. Axis x = points towards the intersection of the equator and the Greenwich meridian, y is towards the intersection of the equator and 90°E, and z = is towards the north pole.

Representation of vectors

In the following, vector **a** is represented $\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$

with components in directions east, north and up.

In printed text, vectors are bold. In handwritten text, vectors are underlined thus $\underline{a}$

Magnitude of vector **a** is represented by
a or $|\mathbf{a}| = \sqrt{a_x^{\;2} + a_y^{\;2} + a_z^{\;2}}$

Unit vector in same direction as **a** $\quad \hat{\mathbf{a}} = \begin{pmatrix} a_x/|\mathbf{a}| \\ a_y/|\mathbf{a}| \\ a_z/|\mathbf{a}| \end{pmatrix}$

Basic vector operations

Vector addition

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} + \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{pmatrix}$$

Vector dot product **a.b** = $ab\cos\theta$ where θ is the angle between **a** and **b**

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \bullet \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x.b_x + a_y.b_y + a_z.b_z$$

Vector cross product

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - b_y a_z \\ a_z b_x - b_z a_x \\ a_x b_y - b_x a_y \end{pmatrix}$$

Vectors and Orientation data

Planes and poles

Plunge (P) and trend (T) of pole from strike (S) and dip (D) of plane

P = 90-D   T = S - 90° or S + 270°

Strike (S) and dip (D) of plane from plunge (P) and trend (T) of pole

D = 90-P   S = T + 90° or T - 270°

Direction cosines

Vector components of a unit vector (direction cosines) (l,m,n) from plunge (P) and trend (T)

l = sin T cos P
m = cos T cos P
n = -sin P

Plunge (P) and trend (T) from direction cosines (l,m,n) (components of unit vector)

$T = \tan^{-1}(l/m)$ *if m is positive or*
$T = \tan^{-1}(l/m) + 180$ *if m is negative*
$P = -\sin^{-1} n$

Plunge (P) and trend (T) from components of a general vector (x,y,z)

$T = \tan^{-1}(x/y)$ *if m is positive or*
$T = \tan^{-1}(x/y) + 180$ *if m is negative*
$P = -\sin^{-1}(z/\sqrt{(x^2+y^2+z^2)})$

Vector components of a pole to a plane (direction cosines) (l,m,n) from strike (S) and dip (D)

l = -cos S sin D
m = sin S sin D
n = -cos D

Strike (S) and dip (D) from direction cosines of a pole (l,m,n)

$T = \tan^{-1}(l/m) + 90$ *if m is positive or*
$T = \tan^{-1}(l/m) + 270$ *if m is negative*
$P = -\cos^{-1} n$

Statistics of orientation data

Statistics for a set of unit vectors: $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \hat{\mathbf{a}}_3 \ldots \hat{\mathbf{a}}_n$

Vector sum $\mathbf{R} = (\hat{\mathbf{a}}_1 + \hat{\mathbf{a}}_2 + \hat{\mathbf{a}}_3 + \ldots\ldots \hat{\mathbf{a}}_n)$

Resultant $R = |\mathbf{R}|$

Vector mean $\mathbf{r} = \overline{\mathbf{R}} = \mathbf{R}/n$

Mean resultant $r = |\mathbf{r}| = R/n$

Direction cosine matrix of a set of unit vectors

The matrix $\begin{pmatrix} \sum l^2 & \sum lm & \sum ln \\ \sum lm & \sum m^2 & \sum mn \\ \sum ln & \sum mn & \sum n^2 \end{pmatrix}$

has eigenvectors $\mathbf{E_1}$, $\mathbf{E_2}$, $\mathbf{E_3}$, and eigenvalues $E_1 \le E_2 \le E_3$