Basic measures of stress

Stress Tensor

T = $\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$

where σ_{xx} is normal stress on plane x
and σ_{xy} is shear stress on plane x parallel to axis y
T is symmetric (i.e. $\sigma_{xy} = \sigma_{yx}$)

Principal stresses

$\sigma_1 > \sigma_2 > \sigma_3$ are the normal stresses that operate parallel to the eigenvectors of **T** (the stress axes).

Mean stress

If the magnitudes of the principal stresses are σ_1 σ_2 σ_3 then:
$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$

Differential stress

$\sigma_d = \sigma_1 - \sigma_3$
Maximum shear stress = $\sigma_d/2$

Deviatoric stress

T$_\mathbf{d}$ = $\begin{pmatrix} \sigma_{xx}-\sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}-\sigma_m & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}-\sigma_m \end{pmatrix}$

Stress on a plane

For a plane with unit normal vector vector **x**,
σ = **T.x**
Resolved normal stress σ_n = **σ.x**
If σ is the magnitude of the stress on the plane, σ_n is the magnitude of the normal stress and σ_s is the magnitude of the shear stress, then:
$\sigma^2 = \sigma_n^2 + \sigma_s^2$

Basic measures of strain

Strain in one dimension

Extension (sometimes elongation) $e = (l-l_0)/l_0$
Stretch $S = l/l_0 = 1+e$
Quadratic elongation $\lambda = l^2/l_0^2 = (1+e)^2$
Natural strain $\varepsilon = ln(S) = ln(1+e) = ln(l/l_0)$
where original length is l_0 and new length is l
Engineering shear strain $\gamma = tan\ \psi$
Tensor shear strain $e_s = 0.5\ tan\ \psi$
where angle of shear is ψ

*Strain in 2 dimensions**

Principal strains are designated by subscripts 1 and 3, e.g. principal elongations are $e_1 > e_3$ principal stretches are $S_1=X,\ S_3=Z$
Strain ratio $R_s = S_1/S_3$
Dilation $1+\Delta = S_1S_3$
Fundamental strain equations (Mohr circle)
For a line at an angle θ from the S_1 axis, if $\lambda' = 1/\lambda$ and $\gamma' = \gamma/\lambda$ *then*

$$\lambda' = \frac{\lambda_3' + \lambda_1'}{2} - \frac{\lambda_3' - \lambda_1'}{2}\cos(2\theta)$$

and

$$\gamma' = \frac{\lambda_3' - \lambda_1'}{2}\sin(2\theta)$$

If λ' is plotted against γ' these are the equations of a circle centred at $\lambda = \frac{\lambda_3' + \lambda_1'}{2}$ with radius $\frac{\lambda_3' - \lambda_1'}{2}$

Shear zones
For a simple shear zone with angle of shear ψ, shear strain γ, the extension axis S_1 is inclined to the shear zone boundary with angle θ given by:
$\gamma = \tan(\psi) = 2 / \tan(2\theta)$

Reorientation of lines from strain ellipse
For a line with initial orientation α and orientation after deformation α'
$tan(\alpha-\theta)/tan(\alpha'-\theta') = R_s$
where R_s is the strain ratio, θ is initial clockwise angle of S_1 from x axis; θ' is clockwise angle of S_1 from x axis after deformation.

*Strain in 3 dimensions**

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are $e_1 > e_2 > e_3$ Principal stretches are $S_1=X,\ S_2=Y,\ S_3=Z$
Dilation $1+\Delta = S_1S_2S_3$
Condition for plane strain: $S_2=1;\ S_1S_3=1$
Strain ratios for Flinn plot
$a = S_1/S_2$
$b = S_2/S_3$
Shape parameter $k = a/b$

*Note: the formulas are based on Ramsay, J.G., and Huber, M.I. (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis, Academic Press, London*, but with sign convention modified for clockwise measurement of angles and shear strains