

## Stress-strain relationships

### Elastic strain

Linear stress  $\sigma_n = E \cdot e$

Poisson's ratio  $\nu = -e_1/e_3$

shear stress  $\sigma_s = G \cdot \gamma$

mean stress  $\sigma_m = -K\Delta$

where E = Young's modulus of elasticity;

G = shear modulus of elasticity;

K = bulk modulus of elasticity;

$e_1, e_3$  are principal extensions of sample under uniaxial compression.

Relationships between the elastic moduli

$$G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)}$$

### Brittle Failure

Coulomb fracture criterion

$$\sigma_s = C + \sigma_n \tan\phi$$

where C is a constant and  $\phi$  is the angle of internal friction.

### "Byerlee's law" for movement on existing fractures

$$\sigma_s = 0.85 \sigma_n \quad (\sigma_n < 200 \text{ MPa})$$

$$\sigma_s = 50 + 0.60 \sigma_n \quad (\sigma_n > 200 \text{ MPa})$$

## Strain in 3 dimensions

Principal strains are designated by subscripts 1, 2 and 3, e.g. principal elongations are  $e_1 > e_2 > e_3$

Principal stretches are  $S_1=X, S_2=Y, S_3=Z$

**Dilation**  $1+\Delta = S_1 S_2 S_3$

**Condition for plane strain:**  $S_2=1; S_1 S_3=1$

### Strain ratios for Flinn plot

$$a = S_1/S_2$$

$$b = S_2/S_3$$

Shape parameter  $k = a/b$

### Viscous (Newtonian) flow

Strain rate  $\dot{\epsilon} \propto \sigma_d$

where  $\sigma_d$  is differential stress. Alternatively:

$$\sigma_d = \eta \dot{\epsilon}$$

where  $\eta$  = viscosity.

Specific equation for diffusional creep

$$\dot{\epsilon} = A \sigma_d \exp\left(-E^*/RT\right) d^{-r}$$

where d is the grain size and r is 2 for grain-boundary diffusion and 3 for volume diffusion

### Plastic (Von Mises) Creep

$$\sigma_d = C \quad (\text{constant})$$

### Power Law Creep

$\dot{\epsilon} \propto \sigma_d^n$  where k and n are constants

or more specifically

$$\dot{\epsilon} = A \sigma_d^n \exp\left(-E^*/RT\right)$$

where A and n are constants for the material, E\* is the activation energy, R is the gas constant and T is the absolute temperature.

### Exponential Creep

$\dot{\epsilon} \propto \exp(\sigma_d)$  where k and n are constants

or more specifically:

$$\dot{\epsilon} = A \exp(\sigma_d) \exp\left(-E^*/RT\right)$$

where A is a constant for the material, E\* is the activation energy, R is the gas constant and T is the absolute temperature