The centrifugal force

As we discussed in class, one of the biggest complications to our description of the atmosphere is that our Earth is rotating, and we are rotating with it. (How fast are we going at the equator even if we’re standing still? How fast is our planet going about the Sun?)

The description of velocities in a rotating framework requires care. Take, for example, the case in which we are NOT in a rotating frame of reference and we are observing a ball on a string being whirled around in a circle of radius $r$.

If you were to sketch the velocity of the ball at different points about the circle that it makes you’ll see that the DIRECTION of the velocity changes with time (but not the speed). If the direction of the velocity changes then there MUST BE an acceleration applied. Looking at the diagram below, you can convince yourself that this acceleration MUST ALWAYS BE POINTED TOWARDS THE AXIS OF ROTATION.

The magnitude of the velocity, $V$, of the ball is related to the rotation rate $\Omega$ and the radius, $r$, by $V=\Omega r$.

The units of $\Omega$ are $s^{-1}$ and the units of $r$ are meters. This gives m/s for the velocity.

$\Omega$ is called the ROTATION RATE. It equals $2\pi$ (radians) divided by the time it takes to complete one circle. It is therefore a measure of how many radians are swept out per second.

For example, our Earth rotates about its axis once every 24 hours. Therefore, its rotation rate is $2\pi/(24*3600s)=7.2722\times10^{-5} s^{-1}$

Earth’s rotation rate is therefore $\Omega=7.2722\times10^{-5} s^{-1}$

**QUESTION**: From the above arguments, can you calculate what the velocity $V$ is of someone standing at rest on the equator? How about of someone standing at rest in Edmonton?

**Hint**: the radius in the above relation for $V$ is the distance to the axis of rotation, which is not necessarily the radius of the Earth!

One can show that the magnitude of the acceleration is proportional to the square of the rotation rate times the radius, such that $$dV/dt = -\Omega^2 r$$

There is therefore an acceleration of magnitude $\Omega^2 r$ directed towards the axis of rotation. This acceleration is called the centripetal acceleration. It is caused by the force of the string pulling the ball. If the string were to break, this force would disappear and the ball would fly off in the direction of its linear velocity, $V$, at that time.

Note also that the centripetal acceleration is only seen from a stationary, non-rotating frame of reference.

If we were in the rotating frame of reference (i.e., if we were sitting on top of the ball) then we would feel as if we were motionless. But there would still be the (very real) force of the string on the ball! This force must be balanced in order for us to be motionless.

Therefore, an APPARENT FORCE, the CENTRIFUGAL FORCE, is invoked such that it is equal and opposite to the force caused by centripetal acceleration. In other words, the centrifugal force per unit mass is directed away from the axis of rotation with a magnitude of $\Omega^2 r$. Note that $\Omega^2 r$ is an acceleration, so to calculate the force simply multiply it by the mass in question (your mass, for example!).

**BY ANALOGY** with the above arguments, imagine the ball as being an air parcel in our atmosphere, the radius as being the distance to the axis of rotation of our Earth (which varies with latitude from ~6380 km at the equator to zero at the poles), and the rotation rate being the value given above for Earth.

**QUESTION**: You and I also feel this centrifugal force. What is its magnitude for an 80 kg person at rest in Edmonton? What is its magnitude for the same person on the equator? If that person were to stand on a scale to weigh themselves, what would the difference in their apparent mass be? (**Hint**: weight scales assume a constant
gravitational acceleration of 9.8 m/s\(^2\). Want to lose weight? Go to the equator!!)

\[ \Omega^2 r = g, \text{ or } \Omega = 1.22 \times 10^{-3} \text{ s}^{-1} \]

Since \( V = \Omega r \) this is equivalent to \(~8000\) m/s! You would therefore have to travel at nearly 8 km per second in order to maintain the force balance (weightlessness!). You would travel around the globe once every \( 2\pi/V \) seconds (circumference divided by velocity), or 5128 seconds.

So, you’d go around the world in 85 minutes!! This is very close to the time it takes for the space shuttle to go around the globe. It’s not rocket science… it’s as easy as \( A = B \)!!