Tornadogenesis Resulting from the Transport of Circulation by a Downdraft: Idealized Numerical Simulations

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ABSTRACT

Idealized numerical simulations are conducted in which an axisymmetric, moist, rotating updraft free of rain is initiated, after which a downdraft is imposed by precipitation loading. The experiments are designed to emulate a supercell updraft that has rotation aloft initially, followed by the formation of a downdraft and descent of a rain curtain on the rear flank. In the idealized simulations, the rain curtain and downdraft are annular, rather than hook-shaped, as is typically observed. The downdraft transports angular momentum, which is initially a maximum aloft and zero at the surface, toward the ground. Once reaching the ground, the circulation-rich air is converged beneath the updraft and a tornado develops. The intensity and longevity of the tornado depend on the thermodynamic characteristics of the angular momentum-transporting downdraft, which are sensitive to the ambient low-level relative humidity and precipitation character of the rain curtain. For large low-level relative humidity and a rain curtain having a relatively small precipitation concentration, the imposed downdraft is warmer than when the low-level relative humidity is small and the precipitation concentration of the rain curtain is large. The simulated tornados are stronger and longer-lived when the imposed downdrafts are relatively warm compared to when the downdrafts are relatively cold, owing to a larger amount of convergence of circulation-rich downdraft air. The results may explain some recent observations of the tendency for supercells to be tornadic when their rear-flank downdrafts are associated with relatively small temperature deficits.

1. Introduction and rationale

The association between hook echoes and tornadoes in supercell storms has been well known for several decades (Stout and Huff 1953; van Tassell 1955; Fujita 1958), as has been the relationship between hook echoes and the rear-flank downdrafts (RFDs) of supercells (Browning and Donaldson 1963; Haglund 1969; Fujita 1973, 1975, 1979). Although hook echoes and their associated RFDs have been long surmised as being important in the development of rotation near the ground (Ludlam 1963; Fujita 1975; Burgess et al. 1977; Barnes 1978; Lemon and Doswell 1979), their precise role in the tornadogenesis process remains unclear. A lengthy review of observational, numerical modeling, and theoretical findings pertinent to hook echoes and RFDs recently has been completed by Markowski (2002).

In a companion paper by Markowski et al. (2002), it was observed that the air parcels at the surface within the RFDs of tornadic supercells tend to be more buoyant and potentially buoyant than within RFDs associated with nontornadic supercells. However, it was not known why this tendency was observed. The findings of Markowski et al. (2002) are the motivation for this idealized numerical study investigating the effects of the thermodynamic characteristics of a downdraft on the intensification of vorticity at the surface. These simulations are designed to capture some of the salient features of tornado formation in supercell storms, at least based on the current state of our theoretical and observational understanding.

The idealized simulations are axisymmetric and contain a rotating, moist, buoyant updraft, which is designed to emulate that of a supercell storm. Rotation is initially absent at the surface; that is, vortex lines turn...
Fig. 1. By the time tornadogenesis [and tornadogenesis “failure” (Trapp 1999)] occurs, RFDs and hook echoes often are observed to nearly completely encircle the intensifying vortex. Thus, an axisymmetric assumption may be fairly reasonable by this stage in supercell evolution. (left) Photograph taken shortly after the genesis of a tornado near Dimmitt, TX, on 2 Jun 1995. Observe that cloud erosion (likely a visual manifestation of subsiding air) appears to have completely encircled the tornado from the perspective of the photographer. Photograph courtesy of the National Oceanic and Atmospheric Administration. (right) Radar reflectivity display from Grand Island, NE, on 3 Jun 1980. The hook echo has completely encircled an echo-free region, which was collocated with a tornado at the time of the image (from Fujita 1981).

toward a horizontal orientation near the ground. The environments of supercell storms contain large vertical wind shear, which implies that vortex lines are initially quasi horizontal. It has been shown that supercell updrafts acquire rotation aloft by tilting horizontal vorticity present in the inflow (Rotunno 1981). The horizontal vorticity sometimes may be enhanced by horizontal buoyancy gradients associated with storm-scale precipitation regions (Klemp and Rotunno 1983; Rotunno and Klemp 1985), resulting in rotation at low levels (~1 km above the ground) following tilting of the augmented horizontal vorticity. However, in the absence of pre-existing vertical vorticity at the surface, it has been shown that a downdraft is needed in order for rotation to develop next to the ground (Davies-Jones 1982a,b; Davies-Jones and Brooks 1993; Walko 1993; Wicker and Wilhelmson 1995). Thus, we initiate a downdraft in our idealized simulations, and this downdraft transports rotation from aloft to the surface. The downdraft is driven by an annulus of rainwater, and the downdraft and rainwater are intended to emulate the RFD and hook echo, respectively.

Although observed supercells are not axisymmetric, as the RFD descends in a spiraling manner [photographs of clear slots reveal that RFDs wrap cyclonically around the mesocyclone (Beebe 1959; Moller et al. 1974; Peterson 1976), to the extent that the cloud erosion is associated with descending air], it may eventually nearly encircle the developing vortex (Fig. 1). By this stage, it is possible that the axisymmetric model of the updraft–downdraft system could be more realistic than in the earlier stages of supercell and RFD evolution. Furthermore, radar observations reveal that hook echoes are downward extensions of the rear side of an elevated reflectivity region (e.g., Forbes 1981). Thus, the method of introducing a “cascade” of rainwater in order to generate a downdraft in these numerical experiments also has some observational justification.

Once circulation arrives at the surface within the downdraft, some of the circulation-bearing air parcels are converged and “recycled” by the updraft, resulting in a tornado. It will be shown that the final concentration of vorticity at the surface depends upon the low-level stability, as also shown by Leslie and Smith (1978) in dry axisymmetric experiments. The static stability profile was specified a priori by Leslie and Smith, and a swirling wind velocity was imposed on inflow entering the domain through the lateral boundary.

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1 In order for net cyclonic (anticyclonic) rotation to develop, a supercell must ingest streamwise (antistreamwise) horizontal vorticity (Davies-Jones 1984).

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The term “recycled” is borrowed from Fujita (1975), who described a process of recycling of downdraft air in his “recycling hypothesis”: 1) downdraft air is recirculated into the (developing) tornado; 2) this process results in an appreciable convergence on the back side of the (developing) tornado; 3) the downward transport of the angular momentum by precipitation and the recycling of air into the tornado will create a tangential acceleration required for the intensification of the tornado. Although Fujita’s use of the term recycling may have implied that air rises in the tornado, exits it, is transported back to the ground by the rear-flank downdraft, and reenters the tornado, our use of the term only implies that air parcels, gently ascending at low levels a few kilometers from the storm, enter a downdraft that transports the parcels to the surface, followed by the reentry of the parcels into the updraft by way of the tornado or incipient tornado.
Our work is intended, to some degree, to advance the work of Leslie and Smith. In the experiments herein, the low-level stability evolves in time as evaporation and entrainment within the downdraft alter the low-level thermodynamic characteristics. No swirling wind component is imposed at the inflow boundary. Instead, the downdraft is responsible for the low-level vortex generation, which ultimately is governed by the ambient environmental conditions and precipitation concentration within the rain curtain. In summary, these idealized simulations contain the gross characteristics long known to be associated with tornado formation in supercell storms: 1) in early stages, a rotating updraft, with maximum rotation aloft and no appreciable rotation at the surface; 2) next, the development of a hook echo and RFD; 3) finally, the arrival of rotation at the surface, and subsequent rapid intensification of rotation into a tornado. The design of the experiments is illustrated in Fig. 2.

The idealized numerical simulation design described above is similar to that used by Davies-Jones (2000). Davies-Jones represented a supercell updraft in an axisymmetric model as a Beltrami flow. An annular rain curtain was then imposed, which dragged angular momentum to the surface. Convergence of the angular momentum at the ground subsequently led to the formation of a tornado. The simulations performed herein can be viewed as an extension of Davies-Jones’ work. The effects of stratification and latent heat release are included in the present idealized simulations. Our simulations are intended to investigate effects that cannot be included when approximating a supercell updraft as a Beltrami flow, such as the effect of the thermodynamic characteristics of the angular momentum-transporting downdraft on the subsequent amplification of vorticity at the surface.

The idealized simulations performed herein also share some similarities with the simulations conducted by Das (1983) and Walko (1988). Walko simulated a rotating, moist (but rain free) updraft in its entirety, whereby the conditionally unstable base state was specified using a sounding obtained on a day on which supercell thunderstorms were observed. The updraft rotation was imposed at the beginning of the integration, and the base of the initial vortex extended to the ground. Therefore, stretching of vorticity alone was able to rapidly spawn intense rotation at the surface. Das’ simulations were designed to investigate vortex spinup within the subcloud layer beneath an updraft. The domain was situated entirely within an implied (1.5 km deep) subcloud layer. The initial wind field was in solid body rotation that decreased exponentially to zero at the lower boundary. Precipitation was inserted at the top boundary, with a maximum concentration on the axis decreasing to zero at the lateral boundary. The precipitation was centrifuged toward the lateral boundary by the rotating wind field, and ultimately gave rise to an annular downdraft that was shown to be capable of transporting sufficient angular momentum to the surface for the genesis of a tornado-like vortex. The vortex was sustained by frictional convergence, which maintained a release of latent heat.

The thermodynamic properties of a precipitation-driven downdraft are sensitive to the amount of evaporation; thus, the downdraft properties presumably would be a function of the ambient sounding characteristics, as well as the precipitation concentrations within the rain curtain. In addition to the relationship between RFD thermodynamic characteristics and tornado likelihood, Markowski et al. (2002) found a relatively large correlation between the coldness of RFDs and the ambient (inflow) relative humidity. Relatively warm (i.e., virtual potential temperature deficits <5 K) RFDs were more likely in moist low-level environments than in dry low-level environments. In these simple experiments, we investigate the effects of the ambient lifting condensation level (LCL; a function of the low-level relative humidity) and the precipitation character of the rain curtain on the thermodynamic properties of an angular momentum-bearing downdraft, and ultimately on tornado intensity and longevity. Midlevel storm-relative winds impinging on an updraft also might play an important role for initiating and maintaining a downdraft (Browning and Ludlam 1962; Browning and Donaldson 1963; Browning 1964), but the model symmetry precludes a study of these effects. Furthermore, it is believed that determination of the dominant RFD forcings as a function of supercell type, evolutionary stage, and location within the supercell is beyond the scope of what presently can be done. It is believed that more observational data are needed in order to determine whether or not the most sophisticated cloud models have reasonable representations of condensate types and spatial distributions, as well as realistic mixing and entrainment in supercell-storm flows. Therefore, we feel that the relatively simple axisymmetric approach has some advantages. Moreover, the use of an axisymmetric model greatly reduces the degrees of freedom, allowing us to isolate just a few physical processes that seem to be especially relevant in tornadogenesis.

The experiments are designed to address the following questions. 1) What are some of the conditions that allow a rain curtain to reach the ground such that relatively small temperature deficits are observed at the surface in the associated downdraft, 2) what is the effect of the downdraft characteristics on the ensuing tornadogenesis, and 3) how is that effect produced? It is emphasized that the goal of the experiments is to simulate tornadogenesis, rather than tornadoes. For example, complexities related to turbulence parameterization in tornadoes (e.g., Lewellen et al. 2000) are ignored. The simple experiments are undertaken in order to obtain some physical interpretation of some recent observations (Markowski et al. 2002) and suggest possible
Fig. 2. Idealized axisymmetric simulations design. A moist, rotating updraft (rotation is imposed) free of rainwater is generated along the axis (light gray shading denotes the cloud water field). Once an approximately steady state is achieved, rainwater is introduced aloft on the periphery of the updraft within the dark gray region (rainwater is imposed by way of a production term $P_r$; the details appear in section 2b). The resulting precipitation-driven downdraft transports angular momentum toward the ground. A schematic vortex line is shown before (black solid line) and after (black dashed line) the formation of the downdraft. The boundary conditions are indicated at their respective locations, with $\psi$ indicating a scalar. All other variables are defined as in the appendix. As defined by Klemp and Wilhelmson (1978), $c_*$ is an assumed intrinsic phase speed ($40 \text{ m s}^{-1}$ assumed) of the dominant gravity wave modes moving out through the lateral boundary.

fruitful routes for further exploration. In the next section, the techniques are outlined. In section 3, the simulation results are presented. In section 4, interpretation and discussion of the findings are provided. Section 5 contains some final remarks.

2. Methods

a. Model description and initial conditions

The model uses an axisymmetric domain that is 15 km deep and extends outward from the center axis to a
radius of 10 km. The prognostic equations are written in cylindrical coordinates \((r, \phi, z)\) and appear in the appendix. The momentum equations are solved on a uniform \(C\) grid (Arakawa and Lamb 1981) using a Klemp–Wilhelmson time splitting scheme with third-order Runge–Kutta time differencing for the large time step (Wicker and Skamarock 1998, 2002). The horizontal and vertical grid resolution is \(\Delta r = \Delta z = 40\) m. A large (small) timestep of 0.6 (0.05) s is used. Fifth-(third-) order spatial differences are used for the horizontal (vertical) advection computations. All other spatial finite differences are second order. No explicit horizontal or vertical numerical filtering is used. A wave–radiation open boundary condition is employed on the large timestep at the lateral boundary (Klemp and Wilhelmson 1978). The upper boundary is rigid, and the horizontal flow at the lower boundary satisfies a semislip condition, assuming a surface drag \(c_D\) of 0.003 (see appendix).

The results of four experiments are presented. The LCL is 1250 m in one pair of experiments (1 and 2), and in the other pair of experiments (3 and 4), the LCL is increased to 2500 m. For each pair of experiments, the precipitation concentration within the rain curtain is varied (details follow in the next section). The LCL and precipitation concentration differences lead to negative buoyancy differences within the precipitation-driven downdrafts, and the effects of these differences on surface vorticity amplification are the focus of this paper.

The soundings used for both pairs of experiments contain approximately 2000 J kg\(^{-1}\) of convective available potential energy (CAPE), and both were constructed using profiles similar to those used by Weisman and Klemp (1982; Fig. 3), where

\[
\overline{\theta}(z) = \begin{cases} 
\theta_u + (\theta_u - \theta_s) \left( \frac{z}{z_u} \right)^m, & z \leq z_u \\
\theta_u \exp \left( \frac{g}{c_p T_u} (z - z_u) \right), & z > z_u,
\end{cases}
\]

(1)

\[
\overline{h}(z) = \begin{cases} 
\min \left[ 1 - (1 - h_{\min}) \left( \frac{z}{z_u} \right)^m, \frac{q_{so}}{q_{so}(\theta)} \right], & z \leq z_u \\
h_{\min}, & z > z_u,
\end{cases}
\]

(2)

where \(\overline{\theta}\) and \(\overline{h}\) are the base-state potential temperature and relative humidity, respectively; \(\theta_s\) is the potential temperature at the surface; \(\theta_u, T_u\) are the potential temperature and absolute temperature at the tropopause; \(z_u\) is the height of the tropopause; \(h_{\min}\) is the minimum upper-level relative humidity; \(q_{so}\) is the mixing ratio at the surface; and \(m\) is a “shape parameter.” The initial profile of base-state specific humidity \(q_s(z)\) is assigned by applying the Clausius–Clapeyron equation to the \(\overline{\theta}\) and \(\overline{h}\) profiles. The sounding parameters appear in Table 1.

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**Table 1. Model sounding parameters.** (Units of \(\theta_u, \theta_s,\) and \(T_u\) are K. units of \(q_{so}\) are g kg\(^{-1}\). Units of \(z_u\) are m.)

<table>
<thead>
<tr>
<th>Expt</th>
<th>(\theta_s)</th>
<th>(q_{so})</th>
<th>(\theta_u)</th>
<th>(T_u)</th>
<th>(m)</th>
<th>(h_{\min})</th>
<th>(z_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>302.0</td>
<td>13.9</td>
<td>349.1</td>
<td>219.1</td>
<td>1.5</td>
<td>0.25</td>
<td>12 000</td>
</tr>
<tr>
<td>3 and 4</td>
<td>308.7</td>
<td>11.2</td>
<td>347.9</td>
<td>217.9</td>
<td>2.1</td>
<td>0.25</td>
<td>12 000</td>
</tr>
</tbody>
</table>
The updrafts are initiated on the model axis by a thermal bubble with a horizontal radius of 5 km and a vertical radius of 1.5 km, centered 1.5 km above the surface. The maximum amplitude of the initial $\theta$ perturbation is 4 K for each sounding. The updrafts acquire rotation from the initial conditions. Many past tornado simulation studies have introduced rotation by way of the lateral (inflow) boundary condition (e.g., Leslie and Smith 1978; Smith and Leslie 1978, 1979; Howells et al. 1988). But these previous experiments simulated only tornadoes, not the parent updraft. The domains were small, with the lateral boundary typically only 1–2 km from the axis, and a body force usually drove the circulation from the domain top, which typically was only 2–3 km above the surface. The initial velocity field was everywhere zero, but the lateral inflow parcels had an azimuthal wind speed that was amplified by stretching as the parcels approached the axis. For the simulations herein, an entire updraft is being simulated; thus, the domain is much larger. Because the domain is much wider than in the past tornado vortex simulations, the region of compensating subsidence is situated entirely within the model domain. Thus, inflow to the updraft does not pass through the lateral boundary. It might seem possible to make the domain narrower so that the compensating subsidence occurs outside of the domain, and so that the parcels ingested by the updraft are those that come through the lateral boundary, thereby transporting angular momentum toward the axis. However, in this configuration, vorticity erupts first near the surface as angular momentum is converged (the convergence associated with the updraft is a maximum near the ground), and the updraft chokes itself shortly into the simulation. A sustained, deep updraft is not possible, since the rotation in a supercell is not maintained, the updrafts acquire an approximately steady state. The steady-state updrafts have maximum vertical velocities of approximately 50–55 m s$^{-1}$ (Fig. 4). Regardless of the sounding characteristics and the size and amplitude of the initial thermal bubble, the steady-state updrafts have radii of 1000–1500 m at low levels and ~750 m at midlevels (note that this is narrower than those in the axisymmetric numerical model). The $\zeta$, that was chosen yields a maximum circulation ($=\zeta_0 \pi r_c^2$) of approximately $8 \times 10^4$ m$^2$ s$^{-1}$, located at $z = 6000$ m for $r \geq r_c$. The radial velocity ($u$) and vertical velocity ($w$) are zero initially.

The initialization using a Rankine combined vortex is somewhat unrealistic because the circulation increases outward to an asymptotic value. In a horizontally quasi-homogeneous storm environment, the circulation an infinite distance away from an updraft is nearly zero. For a clockwise-turning hodograph (helical inflow environment), updrafts are predominantly cyclonic and downdrafts are anticyclonic. In contrast, as was noted by one of the anonymous reviewers, the initial Rankine combined vortex causes downdrafts to be predominantly cyclonic, and makes vortex intensity less sensitive to the trajectories of the parcels that enter it.

Approximately 900 s after the thermal bubble is released, the updrafts acquire an approximately steady state. The steady-state updrafts have maximum vertical velocities of approximately 50–55 m s$^{-1}$ (Fig. 4). Regardless of the sounding characteristics and the size and amplitude of the initial thermal bubble, the steady-state updrafts have radii of 1000–1500 m at low levels and ~750 m at midlevels (note that this is narrower than observed supercell updrafts). Although the updrafts modify the initial angular momentum distribution during the first 900 s of the simulation, by the time an approximately steady state is achieved, the maximum vertical vorticity values within all of the updrafts are similar to the initial maximum of 0.04 s$^{-1}$ and are located between 5 and 7 km above the ground.

### b. Downdraft generation

No rainwater is permitted to form in the axisymmetric, rotating updrafts. Instead, an annullar curtain of rainwater is imposed by way of a production term once an approximately steady state is achieved at 900 s. The negative buoyancy due to the rainwater initiates a downdraft that advects angular momentum downward, as in the Das (1983) and Davies-Jones (2000) experiments. Rainwater was not allowed to form “naturally” so that downdraft formation and location could be controlled.
Fig. 4. Steady-state updrafts in expts 1–4 at $t = 900$ s. Regions where cloud water mixing ratio exceeds 0.1 g kg$^{-1}$ are shaded gray. Radial ($u$) and tangential ($v$) velocities are contoured at 4 m s$^{-1}$ intervals, vertical velocity ($w$) is contoured at 8 m s$^{-1}$ intervals, and circulation ($\Gamma$) is contoured at 10 000 m$^2$ s$^{-1}$ intervals. Dashed contours indicate negative values. Units on the axes are km.
Updraft maintenance issues also arise when rainwater forms within the updraft because there are no mean storm-relative winds to evacuate hydrometeors from the updraft in an axisymmetric model.

The rainwater production \( P_r \) is given by (also see the appendix and Fig. 2)

\[
P_r(r, z, t) = \begin{cases} P_{ro} R_p(r) Z_p(z), & t \geq 900 \text{ s} \\ 0, & \text{otherwise} \end{cases}
\]

where

\[
R_p(r) = \begin{cases} 1 - \left( \frac{r - r_p}{r_{pr}} \right)^4, & r_{pr} - r_p \leq r \leq r_{pr} + r_p, \\ 0, & \text{otherwise} \end{cases}
\]

\[
Z_p(z) = \begin{cases} \cos \left( \frac{\pi |z - z_p|}{z_p} \right), & z_{pr} - z_p \leq z \leq z_{pr} + z_p, \\ 0, & \text{otherwise} \end{cases}
\]

where \( r_p \) and \( z_p \) are the horizontal and vertical radii of the rainwater production zone, and \( r_{pr} \) and \( z_{pr} \) are the coordinates representing where the rainwater production zone is centered. For all four experiments, \( z_{pr} = 250 \text{ m}, z_p = 3000 \text{ m}, r_{pr} = 500 \text{ m}, \) and \( r_p = 1750 \text{ m} \). In other words, beginning at \( t = 900 \text{ s}, P_r \) generates an annular curtain of rain centered 3000 m above the ground, having a radius of 1750 m and a width of 1000 m.\(^3\) The value of \( r_{pr} \) was chosen so that the rain curtains would descend along the flank of the updraft, as is typically observed. The rain curtain was not introduced at a higher altitude because this lead to unrealistically excessive recycling of rainwater into the updraft. In a real supercell, the rain curtain that constitutes the hook echo falls near the storm summit (Forbes 1981), down the rear side of the updraft, where inflow (that could advect the precipitation toward the updraft axis) is typically weaker than on the forward side of the updraft, where inflow typically is strongest (i.e., the inflow of real supercells is not symmetric with respect to the updraft). In the axisymmetric model, radial inflow is symmetric about the axis and probably unrealistically strong in the region inside the rain curtain. It also is worth noting that the radius of the rain curtain in the model is smaller than a supercell hook echo in proportion to the smallness of the model updraft radius compared to a supercell updraft.

The value of \( P_{ro} \), the maximum amplitude of the precipitation production, was varied in two pairs of experiments. In experiments 1 and 3, \( P_{ro} \) was specified to be \( 2 \times 10^{-4} \text{ g kg}^{-1} \text{ s}^{-1} \), which leads to a maximum rainwater concentration of approximately 4–5 g kg\(^{-1}\) within the rain curtain. In experiments 2 and 4, \( P_{ro} \) was specified to be \( 4 \times 10^{-4} \text{ g kg}^{-1} \text{ s}^{-1} \), which leads to a maximum rainwater concentration of approximately 8–10 g kg\(^{-1}\) within the rain curtain (Table 2).

Table 2. Experiment parameters and descriptions. (Units of LCL height are m and units of \( P_{ro} \) are g g\(^{-1}\) s\(^{-1}\).)

<table>
<thead>
<tr>
<th>Expt</th>
<th>LCL</th>
<th>( P_{ro} )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1250</td>
<td>( 2 \times 10^{-4} )</td>
<td>Low LCL; rain curtain has small rainwater concentration.</td>
</tr>
<tr>
<td>2</td>
<td>1250</td>
<td>( 4 \times 10^{-4} )</td>
<td>Low LCL; rain curtain has large rainwater concentration.</td>
</tr>
<tr>
<td>3</td>
<td>2500</td>
<td>( 2 \times 10^{-4} )</td>
<td>High LCL; rain curtain has small rainwater concentration.</td>
</tr>
<tr>
<td>4</td>
<td>2500</td>
<td>( 4 \times 10^{-4} )</td>
<td>High LCL; rain curtain has large rainwater concentration.</td>
</tr>
</tbody>
</table>

\(^3\) The rain curtain is deformed by the flow; therefore, the 1000-m width is only approximate. In some parts of the subcloud layer, the curtain is less than 1000 m wide. Near the surface, divergence within the downdraft causes the rain curtain width to exceed 1000 m slightly.

The precipitation-driven downdraft advects angular momentum toward the surface where, upon reaching the ground, some of the angular momentum is converged by the updraft, resulting in tornado formation (Fujita’s Recycling Hypothesis). During the descent, evaporation and entrainment occur. The effects of the ambient sounding and rain curtain characteristics on the near-ground thermodynamic properties of the downdraft, and ultimately, on the near-ground intensification of vorticity, are discussed in the next section.

3. Results

Approximately 7–9 min after rainwater is introduced, the precipitation-driven downdraft, bearing high-angular momentum air parcels from aloft, reaches the surface. Upon reaching the ground, some of the high-angular momentum air parcels experience convergence beneath the updraft, and swirling velocities increase. A vortex is defined as a tornado when the following arbitrary criteria are met at the lowest grid level (20 m): 1) vertical vorticity exceeds 0.5 s\(^{-1}\); 2) azimuthal winds exceed 30 m s\(^{-1}\). Time series of azimuthal wind speed and maximum virtual temperature fluctuation at the lowest grid level are presented for all of the experiments in Fig. 5. A sequence of figures is used to document the evolution of the rain curtain, associated downdraft, and tornado in experiment 1 (Figs. 6–8). For the other experiments (2–4), model fields are presented shortly after the time of tornadogenesis (Figs. 9–11). Tables 3 and 4 summarize the characteristics of the tornadoes and downdrafts, respectively.

In experiment 1, by \( t = 1200 \text{ s} \) (300 s after rainwater is introduced; Fig. 6), the downdraft associated with the rain curtain has maximum downward vertical velocities exceeding 8 m s\(^{-1}\). Large circulation descending in the
Fig. 5. Time series of maximum virtual potential temperature fluctuation ($\theta'_v$; solid, K) and maximum azimuthal wind speed ($v$; dashed, s$^{-1}$) at the lowest model grid level (20 m) within the region $r \leq 2000$ m.
The ranges of $u$ (m s$^{-1}$) at the lowest grid level (20 m) within the region $r \leq 2000$ m are approximately 2.5 K. By $t = 1390$ s, the annular rain curtain and downdraft arrive at the surface, and significant angular momentum reaches the surface shortly thereafter (Fig. 7). Radial inflow intensifies near the surface, advecting angular momentum toward the axis, and near $t = 1475$ s, a tornado forms (Fig. 8). A condensation funnel extends to within 500 m of the ground, and a region of anticyclonic vertical vorticity surrounds the cyclonic tornado (circulation contours can be viewed as a “pseudo-downdraft” for the radial and vertical components of vorticity; i.e., vortex lines are embedded in constant-circulation surfaces). Near the time of tornadogenesis, the maximum $\theta_v$ deficits at the surface (located within the rain curtain) are approximately 4.5 K, but near the axis and tornado, deficits are only approximately 2.5 K (Fig. 8; Table 3). The most buoyant downdraft parcels supplying circulation to the tornado also are associated with small equivalent potential temperature ($\theta_v$) deficits. The positive $\theta_v$ anomaly inside the rain curtain (Fig. 8; this feature also develops in experiment 2) arises from dry adiabatic descent that has been dynamically forced by the intensification of low-level rotation. That is, this dynamic feedback to the thermodynamically imposed downdraft possibly may be considered an axisymmetric version of the “occlusion downdraft” identified by Klemp and Rotunno (1983) in three-dimensional numerical simulations of supercell thunderstorms. The vortex develops a two-celled structure, and the maximum tangential velocity is 74 m s$^{-1}$. The tornado does not dissipate until approximately $t = 1800$ s (approximately 300 s duration), by which time the parent updraft aloft has collapsed due to the downward pressure gradient force associated with excess pressure deficit (less than 30 mb) at low levels.

In experiment 2, the rain curtain, having approximately twice the rainwater concentration as that in experiment 1, is associated with more substantial $\theta_v$ deficits at the surface (3–6 K). The precipitation annulus reaches the surface by $t = 1325$ s (not shown). Angular momentum begins spreading toward the axis along the ground shortly thereafter, and at approximately $t = 1400$ s, a tornado forms (Fig. 9). Tornadogenesis occurs earlier in the model integration compared to experiment 1, owing to the larger amount of precipitation available to evaporatively chill the air; thus, a significant downdraft forms more quickly than in experiment 1, resulting in a more rapid downward transport of angular momentum toward the surface. The vortex is two-celled, as in experiment 1, and has a maximum axial downdraft of approximately 12 m s$^{-1}$ at a height 150 m above the surface. The maximum swirling velocity is approximately 55 m s$^{-1}$ (Table 3), which is considerably weaker than in experiment 1. The tornado persists for 285 s.

The rain curtain in experiment 3 has the same precipitation concentration as that in experiment 1, but the LCL is twice as high. The drier low levels through which the rain curtain descends promote a colder downdraft than those in experiments 1 and 2, owing to a larger amount of evaporational cooling. The $\theta_v$ deficits in the precipitation-driven downdraft range from 5.4–6.8 K at the surface at the time that a tornado forms, which is near $t = 1475$ s (Tables 3 and 4; Fig. 10). As in the previous experiments, tornadogenesis occurs as high-angular momentum air reaching the ground in the downdraft is concentrated beneath the axisymmetric updraft on the axis. The maximum swirling velocity (39 m s$^{-1}$) is considerably smaller than that observed in experiment 2, and the tornado duration also is much shorter (Table 3; Fig. 5).

The angular momentum–bearing downdraft in experiment 4 is the coldest, owing to the high LCL and the imposition of a rain curtain similar to that introduced in experiment 2. At the surface, $\theta_v$ deficits range from 5.8–8.0 K at approximately the time that the tornado forms (near $t = 1395$ s). The tornado persists for the shortest duration of all the experiments (<120 s) and is associated with swirling velocities of approximately 35 m s$^{-1}$ at the time that the vortex is most intense (Fig.

### Table 3. Summary of simulated tornado characteristics. The tornado duration is as defined in section 3. The largest radial inflow speed (m s$^{-1}$) is $u_{max}$, the largest tangential wind speed (m s$^{-1}$) is $u_{tmax}$, the largest vertical vorticity (s$^{-1}$) is $\zeta_{zmax}$, the largest circulation (m$^2$ s$^{-1}$), is $\Gamma_{zmax}$, the largest vertical velocity (m s$^{-1}$) is $w_{zmax}$, and the largest pressure deficit (mb) is $p'_{zmax}$. Values of $u_{max}$, $u_{tmax}$, $\zeta_{zmax}$, and $\Gamma_{zmax}$ are at the lowest model level (20 m). Values of $w_{max}$ and $p'_{max}$ are below 2 km (and are associated with the vortices).

<table>
<thead>
<tr>
<th>Exp</th>
<th>Tornadogenesis (s)</th>
<th>Tornado duration (s)</th>
<th>$u_{max}$</th>
<th>$u_{tmax}$</th>
<th>$\zeta_{zmax}$</th>
<th>$\Gamma_{zmax}$</th>
<th>$w_{zmax}$</th>
<th>$p'_{zmax}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1476</td>
<td>288</td>
<td>−39.4</td>
<td>74.1</td>
<td>3.06</td>
<td>$6.2 \times 10^4$</td>
<td>32.9</td>
<td>−35.5</td>
</tr>
<tr>
<td>2</td>
<td>1401</td>
<td>285</td>
<td>−34.7</td>
<td>55.5</td>
<td>1.61</td>
<td>$6.3 \times 10^4$</td>
<td>26.3</td>
<td>−24.3</td>
</tr>
<tr>
<td>3</td>
<td>1476</td>
<td>144</td>
<td>−18.3</td>
<td>38.9</td>
<td>1.30</td>
<td>$6.7 \times 10^4$</td>
<td>24.1</td>
<td>−25.9</td>
</tr>
<tr>
<td>4</td>
<td>1395</td>
<td>117</td>
<td>−17.4</td>
<td>35.5</td>
<td>0.99</td>
<td>$7.1 \times 10^4$</td>
<td>26.0</td>
<td>−25.1</td>
</tr>
</tbody>
</table>

### Table 4. Surface thermodynamic data at the time of tornadogenesis. The ranges of $\theta_v'$ (K) and $\theta_t'$ (K) values span the min and max values at the lowest grid level (20 m) within the region $r \leq 2000$ m.

<table>
<thead>
<tr>
<th>Exp</th>
<th>$\theta_v'$ limit</th>
<th>$\theta_t'$ limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−4.4 to −2.5</td>
<td>−4.1 to 0.1</td>
</tr>
<tr>
<td>2</td>
<td>−6.2 to −3.1</td>
<td>−6.7 to 0.0</td>
</tr>
<tr>
<td>3</td>
<td>−6.8 to −5.4</td>
<td>−9.4 to 0.3</td>
</tr>
<tr>
<td>4</td>
<td>−8.0 to −5.8</td>
<td>−9.9 to 0.8</td>
</tr>
</tbody>
</table>
Fig. 6. Meridional cross sections of radial (u), tangential (v), and vertical velocity (w), pressure fluctuations (p'), virtual potential temperature fluctuation (\(\theta'_v\)) and circulation (\(\Gamma\)) for expt 1 at \(t = 1200\) s (300 s after rainwater is introduced). The light gray region indicates where \(q_c > 0.1\) g kg\(^{-1}\), and the dark gray region indicates where \(q_r > 2\) g kg\(^{-1}\). The abscissa range is \(0 \leq r \leq 2\) km and the ordinate range is \(0 \leq z \leq 2\) km. Velocity components are contoured at 4 m s\(^{-1}\) intervals, pressure is contoured at 1-mb intervals, virtual potential temperature is contoured at 1-K intervals, and circulation is contoured at 10 000 m\(^2\) s\(^{-1}\) intervals. Negative contours are dashed.
Fig. 7. As in Fig. 6, but for $t = 1390$ s (−90 s before tornadogenesis).
Fig. 8. As in Fig. 6, but for $t = 1480$ s (shortly after tornadogenesis).
FIG. 9. As in Fig. 6, but for expt 2 at $t = 1410$ s (shortly after tornadogenesis).
Fig. 10. As in Fig. 6, but for expt 3 at $t = 1480$ s (shortly after tornadogenesis).
11). The radius of maximum winds within the vortex also is noticeably larger than in experiments 1–3.

4. Discussion of model results

a. Vortex intensification and maintenance

The results of the previous section underscore the role of the thermodynamic characteristics of RFDs (emulated by annular downdrafts in these idealized simulations) on the ensuing intensification of vorticity at the ground. The numerical simulation findings—that vortices are more intense and persistent as the buoyancy of the downdraft parcels increases (Table 4)—is consistent with the observational findings of Markowski et al. (2002). The simulations also are consistent with the observation that high boundary layer relative humidity values (i.e., low LCL height and small surface dewpoint depression) are associated with relatively warmer RFDs and more significant tornadogenesis than environments of relatively low boundary layer relative humidity. Furthermore, the precipitation character of the downdraft responsible for transporting circulation to the surface was shown to be of comparable importance to tornado intensity and longevity in the simple experiments. Markowski et al. (2002) also found that surface \( \theta_e \) values were larger in the RFDs associated with tornadic supercells compared to the RFDs associated with nontornadic supercells. In the present simulations, the maximum \( \theta_e \) deficits increased with decreasing vortex strength in experiments 1–4, but the minimum \( \theta_e \) deficits in the RFDs were only slightly different among the experiments (Table 4).

Although the definition of a tornado is somewhat subjective (e.g., if the vertical vorticity criterion arbitrarily used herein was increased to 1.00 s\(^{-1}\), then no tornado formed in experiment 4), in some additional experiments not presented, particularly cold (e.g., minimum \( \theta_e \) deficits >7 K) downdrafts led to the formation of only broad, weakly swirling flows at the ground that likely would not satisfy any reasonable tornado criteria. In these cases, the vortices at the surface resembled the mesocyclones that Markowski et al. (2002) observed at the ground in the large majority of nontornadic supercells sampled.

Although environmental (base state) convective inhibition (CIN) values were similar in all four experiments (~40 J kg\(^{-1}\)), the low-LCL cases had lower levels of free convection (LFCs; Fig. 3). Lowering of the LCL typically cannot occur without an attendant lowering of the LFC unless CIN is increased substantially (if CIN becomes too large, then a surface-based storm cannot develop at all). Thus, it is possible that the argument for a low LFC being propitious for significant tornadoes (e.g., J. Davies 2001, personal communication) may not differ fundamentally from the argument for a low LCL favoring significant tornadoes (e.g., Rasmussen and Blanchard 1998). In some recent three-dimensional numerical simulations of supercells (the spatial resolution was too coarse to resolve tornadoes), McCaul and Cohen (2000) and McCaul and Weisman (2001) reported that a low LCL and high LFC height may be optimal for strong updrafts, presumably as long as environmental CIN is not excessive. It is entirely possible that the LCL and LFC height could have effects on supercells on scales larger than the scale at which vorticity is amplified into a tornado.

To explore why the vortex intensity increased with downdraft temperature most simply, it may be beneficial to consider the inviscid, axisymmetric vertical vorticity equation

\[
\frac{\partial \xi}{\partial t} = -\frac{u}{\sigma r} \frac{\partial \xi}{\partial r} - \frac{w}{\sigma} \frac{\partial \xi}{\partial z} + \frac{\xi}{\sigma r} + \frac{\partial w}{\partial z},
\]

where \( \xi = \partial v/\partial r + v/r \) is the vertical vorticity and \( \xi = -\partial u/\partial z \) is the radial vorticity. Azimuthal vorticity (which can be generated baroclinically) cannot be tilted to produce vertical vorticity in the axisymmetric model (this result is not true in three dimensions). The terms on the right-hand side of (9) are horizontal advection, vertical advection, tilting of radial vorticity into the vertical, and vertical stretching, respectively.

Calculation of the forcing terms in (9) reveals that the most significant differences between the cases with downdrafts having relatively small temperature deficits (e.g., experiment 1) and those with downdrafts having relatively large temperature deficits (e.g., experiment 4) arise in the stretching term (Fig. 12). As the static stability increases (owing to larger \( \theta_e \) deficits within the RFD), vertical motions are inhibited; thus, radial convergence is weaker and the local tangential wind velocity is lessened for a given angular momentum. Leslie and Smith (1978) reached a similar conclusion in their dry vortex simulations. In the experiments herein, maximum swirling velocities actually increased as the angular momentum reaching the surface in the downdraft decreased (Table 3). Angular momentum brought to the surface increased with increasing downdraft temperature deficits, but maximum swirling velocities increased with decreasing downdraft temperature deficits, owing to a closer penetration of angular momentum to the axis, which was associated with more substantial convergence and vorticity stretching. Another perspective is presented in Fig. 13, which displays circulation fields and regions of substantial temperature deficits in a larger region than that depicted in Figs. 6–11.

It also may be appropriate to compare low-level convergence prior to tornadogenesis. Convergence fields are plotted 30 and 60 s prior to tornadogenesis in Fig. 14 for experiments 1 and 4. Viewing convergence fields

\footnote{The term “significant tornado” is subjective, but it is used here to refer to tornadoes that produce damage of F2 intensity or greater, or those that persist longer than 5 min.}
Fig. 11. As in Fig. 6, but for expt 4 at $t = 1400$ s (shortly after tornadogenesis).
Fig. 12. Comparison of convergence ($\partial w/\partial z$), vorticity stretching ($\zeta w/\partial z$), and circulation (Γ) in expts 1 and 4 shortly after tornadogenesis (shaded regions; see legends; units on the $\partial w/\partial z$, $\zeta w/\partial z$, and Γ legends are $s^{-1}$, $s^{-2}$, and $m^2 s^{-1}$, respectively). Virtual potential temperature perturbations (θ_v, 1-K intervals) also are overlaid using dashed contours. The abscissa range is $0 \leq r \leq 500$ m and the ordinate range is $0 \leq z \leq 500$ m.
Fig. 13. Meridional cross section of circulation at $t = 1480$ s in expts 1–4. Regions in which low-level virtual potential temperature deficits exceed 5 K are shaded gray.
before tornadogenesis may give us a more pure sense of the role of the downdraft buoyancy on the intensification of rotation, because once the tornado forms, all kinematic fields are dominated by the tornado, its interaction with the surface, and its intense, largely dynamic vertical pressure gradients. Low-level convergence is significantly stronger when the downdraft has small temperature deficits (experiment 1) compared to the cold downdraft case (experiment 4). The tendency for low-level radial inflow and convergence to increase with decreasing downdraft temperature deficits also is evident by inspection of the angle at which the rain curtains descended to the surface in experiments 1–4 (Figs. 8–11). In experiments 3 and 4, the rain curtains diverge from the axis near the ground, whereas in experiments 1 and 2, the rain curtains are drawn toward the axis at low levels. It may be worth noting that the largest convergence differences arise in approximately the lowest 500 to 750 m; such differences may be difficult to detect observationally except in mobile, ground-based Doppler radar data (e.g., Wurman et al. 1996, 1997; Bluestein and Pazmany 2000), in which sampling presumably occurs at close range and data are available very near to the surface.

Some parcel trajectories also were computed. Trajectories began 30 s prior to tornadogenesis and were terminated 90 s later, and were computed using the 0.6-s time resolution data and bilinear interpolation within grid volumes. The trajectories originated from within the downdraft just inside of the radius at which large circulation (>30 000 m² s⁻¹) was present. Some sense of the tendency for relatively warm downdraft parcels...
Fig. 15. Trajectories in expts 1 and 4 for air parcels originating immediately inside the radius at which large angular momentum was descending to the surface in the annular downdraft. The trajectories commence 30 s prior to tornadogenesis and terminate 90 s later. The black dots along the trajectories indicate parcel locations at the time of tornadogenesis. The abscissa range is $0 \leq r \leq 1000$ m and the ordinate range is $0 \leq z \leq 1000$ m. The contributions to vertical velocity along the bold trajectories from dynamic effects and buoyancy are shown in the insets (height units are m; vertical velocity units are m s$^{-1}$).

Following Weisman and Klemp (1984), the dynamic and buoyant forcings for vertical motion were interpolated along each trajectory, and their contributions to vertical velocity were integrated over time; that is,

\[ w(r, z, t) = w(r_0, z_0, t_0) + \int \left( -c_p \frac{\partial \pi'}{\partial z} \right) dt \]

\[ + \int \left( -c_p \frac{\partial \pi_B}{\partial z} + B \right) dt, \]

where the integrals are taken along each trajectory, and $c_p$ is the heat capacity, $\pi'$ is the mean potential temperature, $\pi' = \pi_D + \pi_B$ is the perturbation Exner function decomposed into pressure perturbation contributions from dynamic ($D$ subscript) and buoyancy effects ($B$ subscript; Schlesinger 1980; Klemp and Rotunno 1983), $B$ is buoyancy, and $w_D$ and $w_B$ are the contributions to vertical velocity from dynamic effects and buoyancy, respectively. The pressure decomposition approach was similar to that used by Trapp and Davies-Jones (1997), whereby $\pi'_B$ was determined (to within a constant, owing to the use of Neumann boundary conditions) by solving a Poisson equation, followed by the determination of $\pi'_D$ as a residual, since the total pressure was prognosed by the model.

Not surprisingly, the dynamic part of the perturbation pressure field (not shown) is dominant just before tornadogenesis and throughout the tornado lifetimes. However, the large dynamic pressure fluctuations that arise are not independent of the buoyancy fields and their associated pressure perturbations. The fields of perturbation pressure due to buoyancy (Fig. 16) reveal that the excessive negative buoyancy in the low-level downdraft in experiment 4 leads to a larger outward-directed radial $\pi'_B$ gradient ($\partial \pi'_B / \partial r$) compared to experiment 1. The differences in the ability of high-angular momentum air to penetrate to the axis in experiments 1 versus 4 can be attributed to these $\partial \pi'_B / \partial r$ differences (and these differences ultimately lead to profound differences in swirling velocity and dynamic pressure perturbations).

\[ \text{As an analogy, the well-known nonhydrostatic vertical pressure gradients of supercell storms, which lead to strong dynamic forcing for vertical motion, cannot develop without a significant buoyancy contribution to vertical motion in the first place (i.e., a buoyant updraft is required to produce the large gradients of vertical velocity that ultimately can give rise to dynamic pressure perturbations).} \]
Fig. 16. Comparison of perturbation pressure due to buoyancy ($p'_B$) shortly after tornadogenesis in expts 1 and 4. (Note that $p'_B$, not $\pi'_B$, is displayed.) The $p'_B$ field (shaded; see legend) is known only to a constant (the units on the $p'_B$ legend are mb). Contours of the radial gradient of $p'_B(\partial p'_B/\partial r)$ also have been overlaid ($2 \times 10^{-3}$ mb m$^{-1}$ intervals; dashed contours negative; contour labels have been scaled by $10^{-5}$ mb m$^{-1}$). The abscissa range is $0 \leq r \leq 2$ km and the ordinate range is $0 \leq z \leq 2$ km.
Another perspective is this: downdraft air parcels can travel two directions upon reaching the surface—inward toward the axis or outward toward the lateral boundary. Air that flows inward must ascend as it nears the axis if low-level convergence is to be maintained, owing to mass continuity. As the temperature deficit of the downdraft air increases, its ability to rise as it approaches the axis is impeded (i.e., it is not easily recycled); therefore, radial convergence and angular momentum transport toward the axis are inhibited, consistent with the fields shown in Fig. 16. Thus, as the downdraft coldness or downdraft angular momentum increase, less downdraft air can flow toward the axis, and consequently, a larger mass of downdraft air must spread away from the axis (air parcels spreading away from the axis do not have to be lifted). The end result is a broader, weaker vortex.

The contributions to vertical velocity from dynamic effects and buoyancy for a couple of trajectories in experiments 1 and 4 are displayed in the insets of Fig. 15. As expected, buoyancy effects contribute toward less negative vertical velocities in experiment 1 compared to experiment 4. Dynamic contributions to vertical velocity are larger in experiment 1 compared to experiment 4, mainly due to the formation of a stronger vortex in experiment 1. (As discussed above, dynamic contributions to vertical velocity clearly are sensitive to buoyancy contributions to vertical velocity, and the vorticity amplification facilitated by the buoyancy contributions to vertical velocity feed back to the dynamic pressure field and its contributions to vertical velocity.)

Although the larger dynamic contributions to low-level vertical velocity in experiment 1 are mainly due to the formation of a stronger vortex, some of the difference between the \( w_{v} \) profiles of experiments 1 and 4 owes to differences in centrifugal forces (= \( M^2/r^3 \), where \( M \) is angular momentum), which inhibit radial inflow and convergence (Davies-Jones 1973; Trapp and Davies-Jones 1997). Forcings for radial inflow were examined along trajectories that entered the tornado near the radius of maximum tangential winds shortly after tornadogenesis (Fig. 17). Trajectories commenced 120 s prior to tornadogenesis and were terminated 60 s after tornadogenesis. The dynamic, buoyant, and centrifugal forcings for radial motion were interpolated along each trajectory (see Fig. 17 inset), and their contributions to radial velocity were integrated over time; that is,

\[
u(r, z, t) = \int \left( -c_p \frac{\partial \pi'}{\partial r} \right) dt + \int \left( -c_p \frac{\partial \pi'}{\partial r} \right) dt + \int \left( \frac{\nu^2}{r} \right) dt, (12)
\]

\[
u = u_v + u_d + u_b + u_c, (13)
\]

where the integrals are taken along each trajectory; and
The retardation of radial inflow by \( u_B \) was larger in experiment 4 compared to experiment 1, due to the larger \( \partial p/\partial r \) at low levels in experiment 4 (Fig. 17). The inhibition of radial inflow by \( u_c \) also was slightly larger in experiment 4 for \( r > 300 \) m, but the \( u_c \) differences became more significant for \( 200 \leq r \leq 300 \) m. The \( u_c \) differences were due to the colder downdraft in experiment 4 transporting larger angular momentum to the surface than the warmer downdraft in experiment 1. Because of the intricate feedbacks between angular momentum concentration and the velocity field (and its associated dynamic forcings), even the relatively simple experimental design is too complex to conclude whether the vortex intensity differences in experiments 1 and 4 are primarily due to differences in buoyancy or primarily due to differences in centrifugal forces. For example, it cannot easily be determined whether a warmer downdraft and smaller \( u_B \) in experiment 4 would have led to larger \( u_B \), thereby negating the effect of the larger \( u_c \) (Fig. 17). From the insets of Figs. 15 and 17, we can be certain of the effects of downdraft negative buoyancy on low-level vertical velocity and convergence, and ultimately vortex intensity. The differences between experiments 1 and 2 (Tables 3 and 4) also indicate that the effects of downdraft negative buoyancy alone are sufficient to lead to significant differences in vortex intensity—similar amounts of angular momentum were transported to the surface at approximately similar radii in those experiments; therefore, centrifugal effects on radial inflow also would have been similar in experiments 1 and 2.

We cannot further quantify the role of centrifugal force differences in leading to the vortex intensity differences between experiments 1 and 4 without either imposing many more restrictions on the simulation design, or else initializing each experiment with a different angular momentum profile, so that both warm and cold downdrafts would transport similar angular momentum to the surface. The latter would lead to a plethora of unavoidable and undesirable consequences (e.g., each updraft would have a different steady state prior to the imposition of the downdraft), making it virtually impossible to draw any meaningful conclusions. We do not yet know if the relationship between RFD coldness and angular momentum transport is the same in observed supercells. This may be a worthwhile topic to explore in the future.

**b. Extension to three dimensions**

An axisymmetric model was used, owing to its simplicity and the high spatial resolution achievable with a manageable number of grid points. But real supercells are not axisymmetric. In real supercells, the RFD typically forms on the upshear flank of the updraft. Thus, downward transport of angular momentum occurs only on the upshear flank, at least initially. Furthermore, baroclinically generated horizontal vorticity within the hook echo, if present, can be converted to vertical vorticity by tilting. However, in the minutes leading up to tornadogenesis, the RFD and hook echo often are observed to nearly encircle the cyclonic circulation (Fig. 1), and the axisymmetric simplification may be more justified.

In observed (three dimensional) supercell environments, significant mean storm-relative flow typically is present (e.g., Maddox 1976); in an axisymmetric model, mean storm-relative flow is precluded by the model symmetry. Therefore, the role of entrainment of midlevel environmental air probably is not adequately represented by an axisymmetric model. In real supercells, the entrainment of potentially cold midlevel air might assume larger importance. On the other hand, the surface thermodynamic properties (e.g., \( \theta_c', \theta_d' \)) in the simulation in which the strongest, longest-lived vortex developed (experiment 1) were similar to the surface thermodynamic characteristics observed in the RFDs associated with prolific tornado-producing supercells (Markowski et al. 2002). Markowski et al. (2002) hypothesized that the entrainment of midlevel environmental air may not be as significant in the RFDs associated with tornadic supercells.

Dynamic forcing for the downdraft was essentially absent in the idealized simulations. In observed supercells, however, there is evidence that both dynamic and thermodynamic forcing for the RFD can be significant, although the dominant forcing seems to be a function of time and location within the RFD, and also probably varies from storm to storm. The dynamic forcing for downdrafts in observed supercells appears to be inherently three-dimensional, which makes its inclusion in an axisymmetric simulation problematic. For example, a downward-directed, nonhydrostatic vertical pressure gradient due to stagnation of relative flow at midlevels would be found on the upshear flank of the updraft (Rotunno and Klemp 1982). Furthermore, the low-level rotation that dynamically forces what Klemp and Rotunno (1983) called an “occlusion downdraft” also is not symmetric with respect to an updraft (Markowski 2002). Although dynamic downdraft forcing could not realistically be included in our idealized simulations, one might expect a downdraft having significant forcing from dynamic effects to be warmer than a downdraft of similar intensity being forced entirely by thermodynamic effects. Thus, for a given downdraft strength and steady-state angular momentum distribution, we hypothesize that as the dynamic contribution to the forcing for a downdraft increases relative to the thermodynamic contribution, the likelihood of a significant tornado developing increases.

**c. Sensitivity studies**

Additional simulations were conducted to explore the sensitivity of the results to the location of the downdraft. 

\( u_B, u_c, \) and \( u_c \) are the contributions to radial velocity from dynamic, buoyancy, and centrifugal effects, respectively.
Increasing (decreasing) the width of the rain annulus ($r_p$) led to more (less) substantial cooling within the downdrafts, leading to weaker (stronger) vortices. Increasing the radius of the rain annulus ($r_p$) and associated downdraft led to slightly weaker vortices, due to the larger horizontal distance along the surface, over which the effects of surface drag are important, that angular momentum had to be advected in order to reach the axis (upon arrival of the angular momentum at the surface in the downdraft). Tornadogenesis also did not proceed as quickly as in the cases presented in section 3, because more time was needed for the high-angular momentum air to spread beneath the updraft axis. When the precipitation was imposed at a smaller radius or higher altitude, a significant precipitation downdraft did not form because most of the rainwater was recycled by the updraft and never reached the ground.

Additional simulations also were conducted in which the surface drag was varied from $c_D = 0.000$ to $c_D = 0.030$. The qualitative differences were maintained between the four experiments, although maximum azimuthal wind speeds varied by up to 10%. Other sensitivities of the simulation results undoubtedly exist, such as the sensitivity to the turbulence parameterization and spatial resolution (e.g., Straka and Rasmussen 1998). A full exploration of these effects is beyond the scope of this paper. Therefore, our results probably should be viewed in a qualitative sense.

Only a relatively small part of the parameter space was explored by the idealized simulations. In the future it may be worth experimenting with a variety of initial azimuthal wind fields, more realistic soundings (e.g., those that are drier in the middle troposphere, like those commonly observed in supercell environments), and more complicated microphysics.

5. Closing remarks

The idealized numerical results may account for the association between relatively warm RFDs and tornado likelihood, intensity, and longevity (and small dewpoint depressions and relatively warm RFDs), as reported by Markowski et al. (2002). The simple simulation results also may partly explain the observations by Trapp (1999) of mesocyclones within tornadic supercells having smaller core radii and being associated with larger vertical vorticity stretching than mesocyclones associated with nontornadic supercells. Furthermore, the sensitivity of the numerical results to the rainwater concentration in the downdraft may account for recent observations by Wakimoto and Cai (2000) that the “... only difference between the Garden City (tornadic) storm and Hays (nontornadic) storm (during the Verification of the Origins of Rotation in Tornadoes Experiment) was the more extensive precipitation echoes behind the rear-flank gust front for the Hays storm.” There also was some indication that the mesocyclone core radius was slightly (~10%) larger in the Hays storm than in the Garden City storm.

The ambient low-level relative humidity profile and precipitation character of an imposed, annular, precipitation-driven downdraft have been shown to influence the thermodynamic characteristics of the downdraft upon reaching the surface. The thermodynamic properties of the downdraft ultimately had a significant effect on the strength and persistence of the vortex arising from the concentration of the angular momentum transported by the downdraft. Downdraft formation was necessary for tornadogenesis owing to the initial absence of circulation at the surface. The warmer downdrafts allowed for larger amounts of stretching of vorticity-rich parcels as they emerged from the downdraft and were recycled by the updraft. In cases in which the downdrafts contained large temperature deficits, the convergence of angular momentum beneath the updraft as downdraft parcels were recycled was weaker than when the downdrafts contained relatively small temperature deficits, owing to low-level vertical velocities that were retarded by the excessive negative buoyancy, as well as increased centrifugal forces in the cold downdraft cases. The results are generalized by way of a schematic illustration in Fig. 18. Remarkably, nearly four decades ago, Ludlam (1963) speculated that “it may be particularly important for the intensification and persistence of a tornado that some of the downdraft air be derived from potentially warm air.”

Three-dimensional numerical simulations of supercells appearing in the literature thus far have not produced the warm RFDs that our idealized simulation results indicate may be propitious for the production of significant tornadoes. This inability probably owes at least partly to inadequate representations of microphysical processes and possibly to insufficient spatial resolution. Many simulations have been performed without the inclusion of ice, which allows for hydrometeors to be distributed over larger areas; thus, cold pools near the updraft tend to be stronger when ice physics is absent. Some recent unpublished simulations, however, have produced warm RFDs at relatively fine horizontal resolutions (~500 m) when ice physics is included (M. Gilmore 2001, personal communication). It is inevitable that less idealized, three-dimensional studies with fairly sophisticated microphysics will be attainable in the near future with horizontal resolutions of 100 m or less (Wicker et al. 2002). It will be worth comparing such future results with those obtained herein.

In addition to environmental conditions (e.g., ambient relative humidity) and processes like entrainment, it would seem that the precipitation species and distribution within the hook echo, in addition to the precipitation concentration (as suggested by the experiments herein), must, to some degree, control the amount of evaporation and resulting thermodynamic characteristics of the RFD. It is speculated that some inferences...
Fig. 18. Schematic generalizing the model results. In this illustration, a downdraft spirals down the periphery of the updraft, approximately 1 km from its center. Upon reaching the ground, relatively cold downdraft parcels spread mostly away from the center, and convergence of angular momentum beneath the updraft is weak, resulting in a short-lived tornado or no tornado at all. But if the downdraft parcels are relatively warm, they more readily spiral toward the center upon reaching the ground, converging angular momentum to a small radius and forming a long-lived, significant tornado. The differences in the degree of angular momentum concentration owe to differences in low-level stability as well as differences in centrifugal forces.

could be made of RFD surface thermodynamic characteristics someday using information about the drop size distribution within hook echoes available from dual-polarization radars [e.g., relatively numerous, small drops may imply colder hooks and RFDs because of increased evaporation potential (Hookings 1965; Kamburova and Ludlam 1966)].

We do not believe that there is a reason why these experimental results would not also be applicable to cases in which a downdraft is not needed for tornadogenesis, that is, cases in which circulation at the surface preexists the updraft, such that the circulation can be concentrated into a tornado by the updraft alone (e.g., the “landspout mechanism”). In other words, we believe that the degree of concentration of near-ground circulation—whether the circulation is present in the ambient environment or must be brought to the surface by a downdraft—depends on the thermodynamic characteristics of the circulation-rich air. Indeed, the Leslie and Smith (1978) experiments, in which only an updraft body force and inflow circulation extending to the surface were imposed, may be directly applicable to such a tornadoogenesis process.

Finally, we reiterate that it may be highly worthwhile to compare near-ground angular momentum versus buoyancy in future observational studies of supercell thunderstorms. Do cold downdrafts generally transport larger angular momentum to the surface? To what extent is tornadoogenesis failure—when adequate angular momentum is present at the surface but cannot be sufficiently converged—a result of excessive negative buoyancy inhibiting convergence versus excessive centrifugal forces arising from too much angular momentum being transported to the surface by cold downdrafts? The cold RFDs found by Markowski et al. (2002) to commonly be associated with nontornadic supercells also tended to have low $\theta_e$ values. Such low $\theta_e$ values may imply that RFD air reaching the surface in nontornadic supercells often originates from higher altitudes than the RFD air reaching the surface in tornado supercells, thereby potentially transporting larger angular momentum to the surface in nontornadic supercells.

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APPENDIX

Model Equations

The prognostic equations for the wind velocity components (u, v, w) are written in cylindrical coordinates (r, φ, z) as follows:

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -u \frac{\partial u}{\partial r} - w \frac{\partial u}{\partial z} + v^2 - c_p \frac{\partial \theta}{\partial r}, \\
&\quad + \frac{1}{r} \frac{\partial}{\partial \phi} \left( r \frac{\partial u}{\partial \phi} \right) - \frac{\sigma_u}{r} + \frac{\partial q_u}{\partial z}, \\
\frac{\partial v}{\partial t} &= -u \frac{\partial v}{\partial r} - w \frac{\partial v}{\partial z} - uv + \frac{1}{r} \frac{\partial (r^2 \sigma_v)}{\partial r} + \frac{\partial q_v}{\partial z}, \\
&\quad + \frac{1}{r} \frac{\partial}{\partial \phi} \left( r \frac{\partial v}{\partial \phi} \right) - \frac{\sigma_v}{r} + \frac{\partial q_v}{\partial z}, \\
\frac{\partial w}{\partial t} &= -u \frac{\partial w}{\partial r} - w \frac{\partial w}{\partial z} - c_p \frac{\partial \theta}{\partial r} + \frac{\partial q_w}{\partial z} + \frac{\partial}{\partial z} \left( \frac{1}{3} \nabla^2 w \right),
\end{align*}
\]

(A1)

where \( \sigma_u = K_u \left( \frac{2}{3} \text{div} + \frac{2u}{r} \right) \)

\( \sigma_v = K_v \left( \frac{2}{3} \text{div} + \frac{2v}{r} \right) \)

\( \sigma_w = K_w \left( \frac{2}{3} \text{div} + \frac{2w}{r} \right) \)

\( \sigma_{\phi \phi} = K_{\phi \phi} \left[ \frac{\partial}{\partial \phi} \left( \frac{u}{r} \right) \right] \)

\( \sigma_{\phi z} = K_{\phi z} \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \right) \)

\( \sigma_{z z} = K_z \left( \frac{\partial w}{\partial z} + \frac{\partial \theta}{\partial z} \right) \)

(A4)

is the axisymmetric divergence and \( K_u \) is the eddy viscosity for momentum.

The total \( \tau \) and \( \theta \) are expanded about an unperturbed base-state \( \overline{\tau} \) and \( \overline{\theta} \) according to

\[
\begin{align*}
\tau(r, z, t) &= \overline{\tau}(z) + \tau'(r, z, t), \\
\theta(r, z, t) &= \overline{\theta}(z) + \theta'(r, z, t),
\end{align*}
\]

(A6)

where the base state is hydrostatic:

\[
\frac{\partial \overline{\tau}}{\partial z} = -\frac{g}{c_p \overline{\theta}}.
\]

(A8)

The pressure equation is obtained by taking the material derivative \( (d/dt) = \partial/\partial t + ud/\partial r + wd/\partial z \) of \( \tau = (p/\rho) \tilde{\rho}' \), and using the compressible continuity equation

\[
\frac{dp}{dt} + \rho \left[ \frac{1}{r} \frac{\partial (ur)}{\partial r} + \frac{\partial w}{\partial z} \right] = 0.
\]

(A9)

where \( \rho \) is the air density, to eliminate \( dp/dt \):

\[
\begin{align*}
\frac{\partial \tau'}{\partial t} &= -c_s^2 \left[ \frac{1}{c_p \overline{\theta}} \frac{\partial}{\partial r} \left( \overline{\theta} \tilde{\rho}' \right) + \frac{\partial}{\partial z} \left( \overline{\theta} \tilde{\rho}' \right) \right] - u \frac{\partial \tau'}{\partial r} \\
- w \frac{\partial \tau'}{\partial z} &- R_s \frac{\partial \tau'}{\partial r} \frac{\partial w}{\partial z} - R_s \frac{\partial w}{\partial z} \frac{\partial \tau'}{\partial t} - \frac{c_s^2}{c_p \overline{\theta}} \frac{d \overline{\theta}}{dt},
\end{align*}
\]

(A10)

where \( c_s = (c_p R_s \overline{\theta}'/\rho) \) is the speed of sound. Following Klemp and Wilhelmson (1978), all but the first term on the right-hand side are neglected.

The prognostic equation for \( \theta \) is

\[
\frac{\partial \theta}{\partial t} = -u \frac{\partial \theta}{\partial r} - w \frac{\partial \theta}{\partial z} + \frac{1}{r} \frac{\partial}{\partial \phi} \left( K \frac{\partial \theta}{\partial r} \right) \\
+ \frac{\partial}{\partial z} \left( K_z \frac{\partial \theta}{\partial z} \right) + H,
\]

(A11)

where \( K \) is the eddy mixing coefficient for heat and \( H \) is the rate of latent heating by condensation of water vapor \( q_w \).

The prognostic equation for \( q_w \) is

\[
\begin{align*}
\frac{\partial q_w}{\partial t} &= -u \frac{\partial q_w}{\partial r} - w \frac{\partial q_w}{\partial z} + \frac{1}{r} \frac{\partial}{\partial \phi} \left( K \frac{\partial q_w}{\partial r} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial q_w}{\partial z} \right) \\
&\quad - S_c + S_e,
\end{align*}
\]

(A12)

where \( S_c \) is the sink of water vapor owing to condensation:

\[
S_c = \frac{c_s'TH}{L_c \overline{\theta}}.
\]

(A13)

where \( L_c \) is the latent heat of vaporization, and \( S_e \) is the source of water vapor owing to evaporation. The latent heating owing to freezing is not included. The evapo-
The eddy viscosity coefficient for momentum was prescribed as a function of the deformation field (Smagorinsky 1963). In the axisymmetric model, it is

\[ K_m = (c\Delta)^2 \left\{ \left( \frac{\partial w}{\partial \zeta} + \frac{\partial u}{\partial \zeta} \right)^2 + 2 \left( \frac{\partial u}{\partial r} + \frac{\partial w}{\partial \zeta} + \frac{u}{r} \right)^2 \right\} \]

\[ + \left( \frac{\partial u}{\partial r} - \frac{v}{r} \right)^2 + \left( \frac{\partial w}{\partial \zeta} \right)^{-1/2}, \]

where \( \Delta = (\Delta r \Delta z)^{1/2} \) and \( c \) is a nondimensional constant chosen to be 0.2 following Deardorff (1972). Also following Deardorff, \( K_m = 3K_m \) is specified.

As stated in section 2a, the lateral boundary is open and the top boundary is rigid. At the lower boundary, \( w = 0 \), but the horizontal flow satisfies the semislip condition

\[ \left( \frac{\partial u}{\partial \zeta}, \frac{\partial v}{\partial \zeta} \right) = -\frac{1}{K_m}(u'w', v'w'), \]

where the surface fluxes are given by

\[ u'w' = c_p \sqrt{u^2 + v^2}u, \quad \text{and} \]

\[ v'w' = c_p \sqrt{u^2 + v^2}v, \]

where the drag coefficient is \( c_p = 0.003 \). The remaining conditions are depicted in Fig. 2.

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