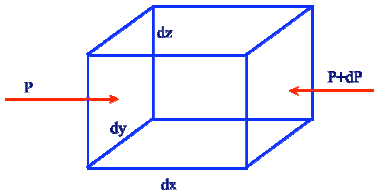


## The Pressure Gradient Force

We've already seen the vertical component of the pressure gradient when we examined hydrostatic balance. However, horizontal variations of pressure also exist and give rise to horizontal pressure gradients.

You can think of the pressure gradient in a given direction as follows.

Consider a volume of air,  $dV = dx dy dz$ . The mass of air in this volume is therefore  $m = \rho dx dy dz$ . Examine the pressure exerted on this volume of air in only the x-direction:



If  $dP=0$  then there are equal and opposite pressures exerted on the two sides of the volume. Each of these sides has an area of  $dy dz$ . Since pressure is force per unit area, the forces exerted on the left and right sides of the box are, respectively,  $P dy dz$  and  $(P+dP) dy dz$ . If  $dP$  is greater than zero, there is an imbalance between the forces and the air would move to the LEFT. If  $dP$  is negative, the air would move to the RIGHT. If we consider this box to be very small, then we can say that  $dP$  is related to the pressure gradient in the x-direction such that

$$dP = \frac{\partial P}{\partial x} dx$$

Therefore, the difference in force in the x-direction (left minus right) is:

$$P dy dz - (P + \frac{\partial P}{\partial x} dx) dy dz = - \frac{\partial P}{\partial x} dx dy dz$$

And from this result, the *force per unit mass in the x-direction* is therefore given by:

$$-\frac{1}{\rho} \frac{\partial P}{\partial x}$$

BY ANALOGY, the gradients in the y and z directions are, respectively,

$$-\frac{1}{\rho} \frac{\partial P}{\partial y} \quad \text{and} \quad -\frac{1}{\rho} \frac{\partial P}{\partial z}$$

These three components make up the **PRESSURE GRADIENT FORCE**.

$$PGF = -\frac{1}{\rho} \left( \frac{\partial P}{\partial x} \hat{i} + \frac{\partial P}{\partial y} \hat{j} + \frac{\partial P}{\partial z} \hat{k} \right)$$

The pressure gradient force drives winds from regions of high pressure to regions of low pressure. Draw a diagram and the above relations to convince yourself that this is the case.

Our bodies use this fact to regulate pressure in our lungs so that air will flow into and out of our lungs. Muscle action expands or decreases lung volume which, respectively, decreases or increases density. By the ideal gas law, these volume changes cause a decrease or increase (respectively) of pressure within our lungs.

The air around us will therefore flow into our lungs when we expand them, and out of them when we compress them. Easy!

Our atmosphere experiences these same forces (recall how surface temperature changes can cause pressure changes). On a very large spatial scale, however, one of the APPARENT forces (the Coriolis force) becomes as large as the pressure gradient force so the flow is not directly from high to low pressure. In the absence of rotation, however, atmospheric flow would be direct from high to low pressure.